

$$\operatorname{div} \vec{E} = \frac{\rho}{\varepsilon}$$

$$\operatorname{rot} \vec{E} = -\mu \frac{\partial}{\partial t} \vec{H} - \vec{J}_m$$

$$\operatorname{div} \vec{H} = \frac{\rho_m}{\mu}$$

$$\operatorname{rot} \vec{H} = \varepsilon \frac{\partial}{\partial t} \vec{E} + \vec{J}$$

$$\begin{aligned} \vec{E} &= \boxed{\vec{E}^H} + \boxed{\vec{E}^E} \\ \vec{H} &= \boxed{\vec{H}^H} + \boxed{\vec{H}^E} \end{aligned}$$

H-Wellen

$$\operatorname{div} \vec{E}^H = \frac{\rho}{\varepsilon}$$

$$\operatorname{rot} \vec{E}^H = -\mu \frac{\partial}{\partial t} \vec{H}^H$$

$$\operatorname{div} \vec{H}^H = 0$$

$$\operatorname{rot} \vec{H}^H = \varepsilon \frac{\partial}{\partial t} \vec{E}^H + \vec{J}$$

$$\frac{\partial \rho}{\partial t} + \operatorname{div} \vec{J} = 0$$

E-Wellen

$$\operatorname{div} \vec{H}^E = \frac{\rho_m}{\mu}$$

$$\operatorname{rot} \vec{H}^E = \varepsilon \frac{\partial}{\partial t} \vec{E}^E$$

$$\operatorname{div} \vec{E}^E = 0$$

$$\operatorname{rot} \vec{E}^E = -\mu \frac{\partial}{\partial t} \vec{H}^E - \vec{J}_m$$

$$\frac{\partial \rho_m}{\partial t} + \operatorname{div} \vec{J}_m = 0$$

$$\begin{aligned}\vec{E} &= \vec{E}^H + \vec{E}^E \\ \vec{H} &= \vec{H}^H + \vec{H}^E\end{aligned}$$

$$\begin{aligned}\vec{E} &= -\mu \frac{\partial}{\partial t} \vec{A}^H - \text{grad} \Phi^H + \text{rot} \vec{A}^E \\ \vec{H} &= \text{rot} \vec{A}^H + \varepsilon \frac{\partial}{\partial t} \vec{A}^E + \text{grad} \Phi^E\end{aligned}$$

$$\begin{aligned}\Phi^H(\vec{r}, t) &= \frac{1}{4\pi\varepsilon} \int_V \frac{\rho(\vec{r}', t')}{\|\vec{r} - \vec{r}'\|} dV' \\ \vec{A}^H(\vec{r}, t) &= \frac{1}{4\pi} \int_V \frac{\vec{J}(\vec{r}', t')}{\|\vec{r} - \vec{r}'\|} dV'\end{aligned}$$

$$\begin{aligned}\Phi^E(\vec{r}, t) &= -\frac{1}{4\pi\mu} \int_V \frac{\rho_m(\vec{r}', t')}{\|\vec{r} - \vec{r}'\|} dV' \\ \vec{A}^E(\vec{r}, t) &= -\frac{1}{4\pi} \int_V \frac{\vec{J}_m(\vec{r}', t')}{\|\vec{r} - \vec{r}'\|} dV'\end{aligned}$$

Elektrischer Dipol (Hertzscher Dipol):

$$\vec{J}(\vec{r}) = I\delta l \delta(x)\delta(y)\delta(z) \vec{e}_z = C_e \frac{4\pi}{jk} \delta(x)\delta(y)\delta(z) \vec{e}_z$$

$$\vec{A}(\vec{r}) = \frac{1}{4\pi} \int_V \frac{\vec{J}(\vec{r}', t')}{\|\vec{r} - \vec{r}'\|} e^{-jk\|\vec{r} - \vec{r}'\|} dV' = \frac{1}{4\pi} \frac{I\delta l}{r} e^{-jkr} \vec{e}_z = C_e \frac{1}{jkr} e^{-jkr} \vec{e}_z$$

$$\vec{e}_z = \cos \vartheta \vec{e}_r - \sin \vartheta \vec{e}_\vartheta$$

$$\vec{H}(\vec{r}) = \text{rot } \vec{A} = \frac{I\delta l}{4\pi} e^{-jkr} \left(\frac{jk}{r} + \frac{1}{r^2} \right) \sin \vartheta \vec{e}_\varphi = C_e \frac{e^{-jkr}}{r} \left(1 + \frac{1}{jkr} \right) \sin \vartheta \vec{e}_\varphi$$

$$\vec{E}(\vec{r}) = \frac{1}{j\omega\epsilon} \text{rot } \vec{H} \quad \text{oder} \quad \vec{E}(\vec{r}) = -j\omega\mu\vec{A} - \text{grad}\Phi$$

Lorentz Eichung:

$$\Phi(\vec{r}) = \frac{1}{-j\omega\epsilon} \text{div} \vec{A} = \frac{C_e}{Z} \frac{e^{-jkr}}{jkr} \left(1 + \frac{1}{jkr} \right) \cos \vartheta$$

$$-\text{grad}\Phi = 2C_e Z \frac{e^{-jkr}}{r} \left(\frac{1}{2} + \frac{1}{jkr} + \frac{1}{(jkr)^2} \right) \cos \vartheta \vec{e}_r$$

$$+ C_e Z \frac{e^{-jkr}}{r} \left(+ \frac{1}{jkr} + \frac{1}{(jkr)^2} \right) \sin \vartheta \vec{e}_\vartheta$$

Elektrischer Dipol (Hertzscher Dipol):

$$\vec{J}(\vec{r}) = C_e \frac{4\pi}{jk} \delta(x)\delta(y)\delta(z) \vec{e}_z$$

$$\vec{H}(\vec{r}) = C_e \frac{e^{-jkr}}{r} \left(1 + \frac{1}{jkr} \right) \sin \vartheta \vec{e}_\varphi$$

$$\vec{E}(\vec{r}) = \boxed{-j\omega\mu\vec{A}} - \text{grad}\Phi$$

$$\begin{aligned} \vec{E}(\vec{r}) = & 2C_e Z \frac{e^{-jkr}}{r} \left(\frac{1}{2} \boxed{\frac{1}{2}} + \frac{1}{jkr} + \frac{1}{(jkr)^2} \right) \cos \vartheta \vec{e}_r \\ & + C_e Z \frac{e^{-jkr}}{r} \left(\boxed{1} + \frac{1}{jkr} + \frac{1}{(jkr)^2} \right) \sin \vartheta \vec{e}_\vartheta \end{aligned}$$

$$\vec{A}(\vec{r}) = C_e \frac{1}{jkr} e^{-jkr} \vec{e}_z$$

Elektrischer Elementardipol (Dipolantenne)

$$\underline{E}_\varphi = \underline{H}_r = \underline{H}_\vartheta = 0$$

$$\underline{E}_\varphi = \underline{C}_e \sin\vartheta \frac{e^{-jkr}}{r} \left(1 + \frac{1}{jkr} \right)$$

$$\underline{E}_r = 2 \underline{C}_e Z \cos\vartheta \frac{e^{-jkr}}{r} \left[\frac{1}{jkr} + \left(\frac{1}{jkr} \right)^2 \right]$$

$$\underline{E}_\vartheta = \underline{C}_e Z \sin\vartheta \frac{e^{-jkr}}{r} \left[1 + \frac{1}{jkr} + \left(\frac{1}{jkr} \right)^2 \right]$$

Magnetischer Elementardipol (Rahmenantenne)

$$\underline{H}_\vartheta = \underline{E}_r = \underline{E}_\vartheta = 0 \quad \underline{E}_\varphi = \underline{C}_m \sin \vartheta \frac{e^{-jkr}}{r} \left(1 + \frac{1}{jkr} \right)$$

$$\underline{H}_r = - 2 \underline{C}_m Y \cos \vartheta \frac{e^{-jkr}}{r} \left[\frac{1}{jkr} + \left(\frac{1}{jkr} \right)^2 \right]$$

$$\underline{H}_\vartheta = -\underline{C}_m Y \sin \vartheta \frac{e^{-jkr}}{r} \left[1 + \frac{1}{jkr} + \left(\frac{1}{jkr} \right)^2 \right]$$

Nahfeld-Näherung

Im Nahfeld gilt:

$$|kr| \ll 1 \Rightarrow e^{-jkr} \approx 1$$

elektrischer Dipol

$$\underline{E}_\vartheta = \underline{H}_r = \underline{H}_\vartheta = 0$$

$$\underline{H}_\varphi = -j C_e \sin \vartheta \frac{1}{kr^2}$$

$$\underline{E}_r = -2 \underline{C}_e Z \cos \vartheta \frac{1}{k^2 r^3}$$

$$\underline{E}_\vartheta = -\underline{C}_e Z \sin \vartheta \frac{1}{k^2 r^3}$$

magnetischer Dipol

$$\underline{H}_\varphi = \underline{E}_r = \underline{E}_\vartheta = 0$$

$$\underline{E}_\varphi = -j C_m \sin \vartheta \frac{1}{kr^2}$$

$$\underline{H}_r = 2 \underline{C}_m Y \cos \vartheta \frac{1}{k^2 r^3}$$

$$\underline{H}_\vartheta = \underline{C}_m Y \sin \vartheta \frac{1}{k^2 r^3}$$

Fernfeld-Näherung

Falls Abstand zum Elementardipol genügend groß, d.h.:

$$|kr| \gg 1$$

elektrischer Dipol

$$\underline{E}_\vartheta = \underline{H}_r = \underline{H}_\vartheta = 0$$

$$\underline{H}_\vartheta = \underline{C}_e \sin \vartheta \frac{e^{-jkr}}{r}$$

$$\underline{E}_r = -2 j \underline{C}_e Z \cos \vartheta \frac{e^{-jkr}}{kr^2}$$

$$\underline{E}_\vartheta = \underline{C}_e Z \sin \vartheta \frac{e^{-jkr}}{r}$$

magnetischer Dipol

$$\underline{H}_\vartheta = \underline{E}_r = \underline{E}_\vartheta = 0$$

$$\underline{E}_\vartheta = \underline{C}_m \sin \vartheta \frac{e^{-jkr}}{r}$$

$$\underline{H}_r = 2 j \underline{C}_m Y \cos \vartheta \frac{e^{-jkr}}{kr^2}$$

$$\underline{H}_\vartheta = -\underline{C}_m Y \sin \vartheta$$

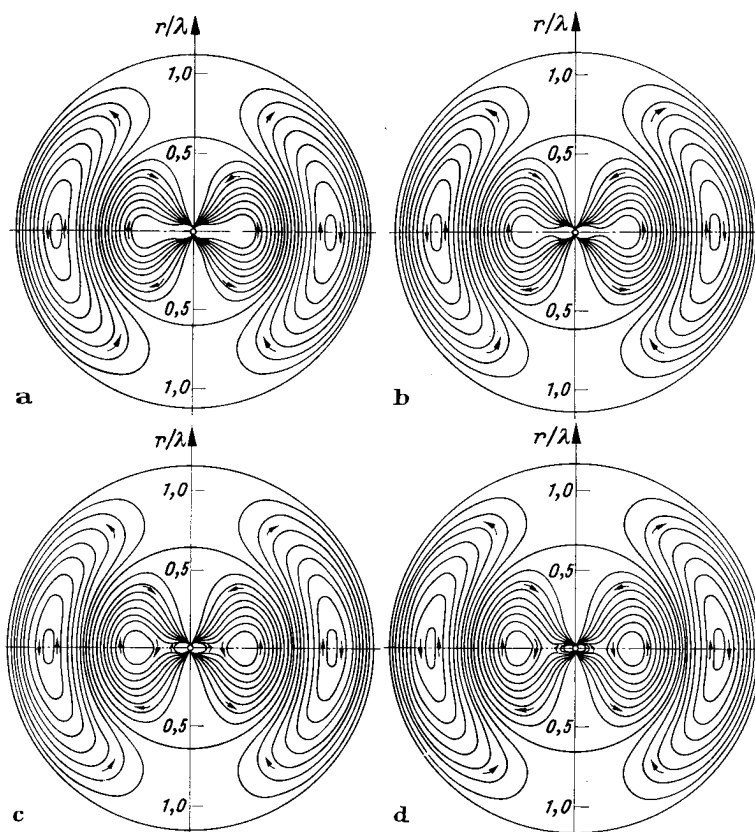


Abb. 6.1/3a–d. Elektrisches Feldbild eines HERTZschen Dipols für verschiedene Zeitpunkte, bezogen auf die Periodendauer T

a $t = 25/64 T$ b $t = 26/64 T$ c $t = 27/64 T$ d $t = 28/64 T$

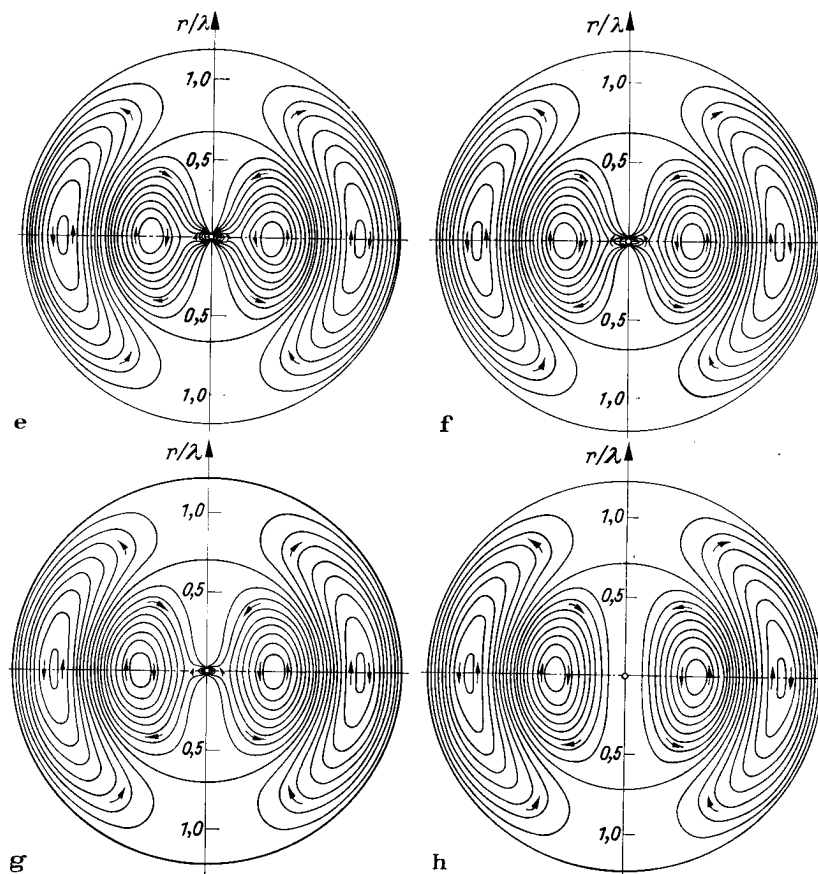
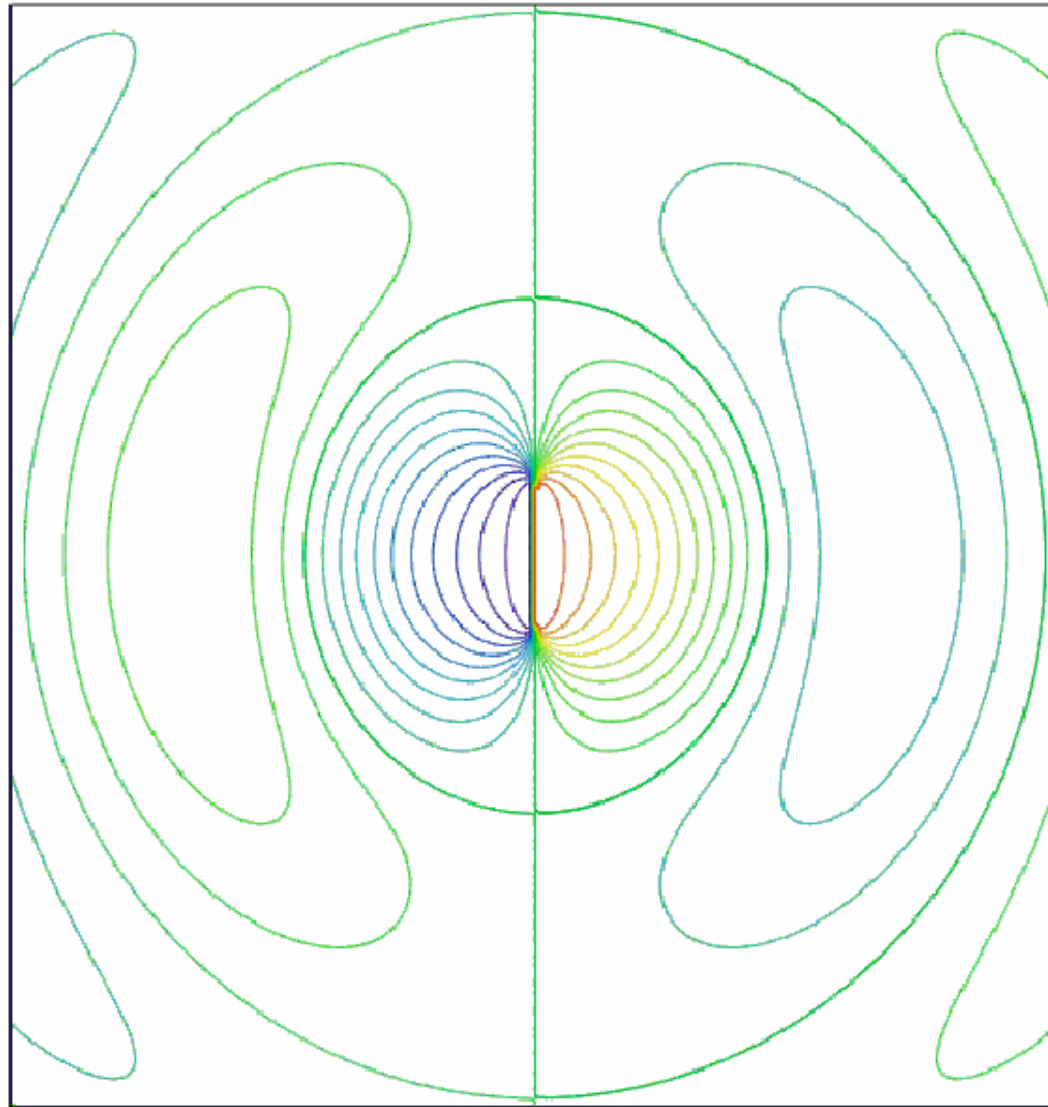
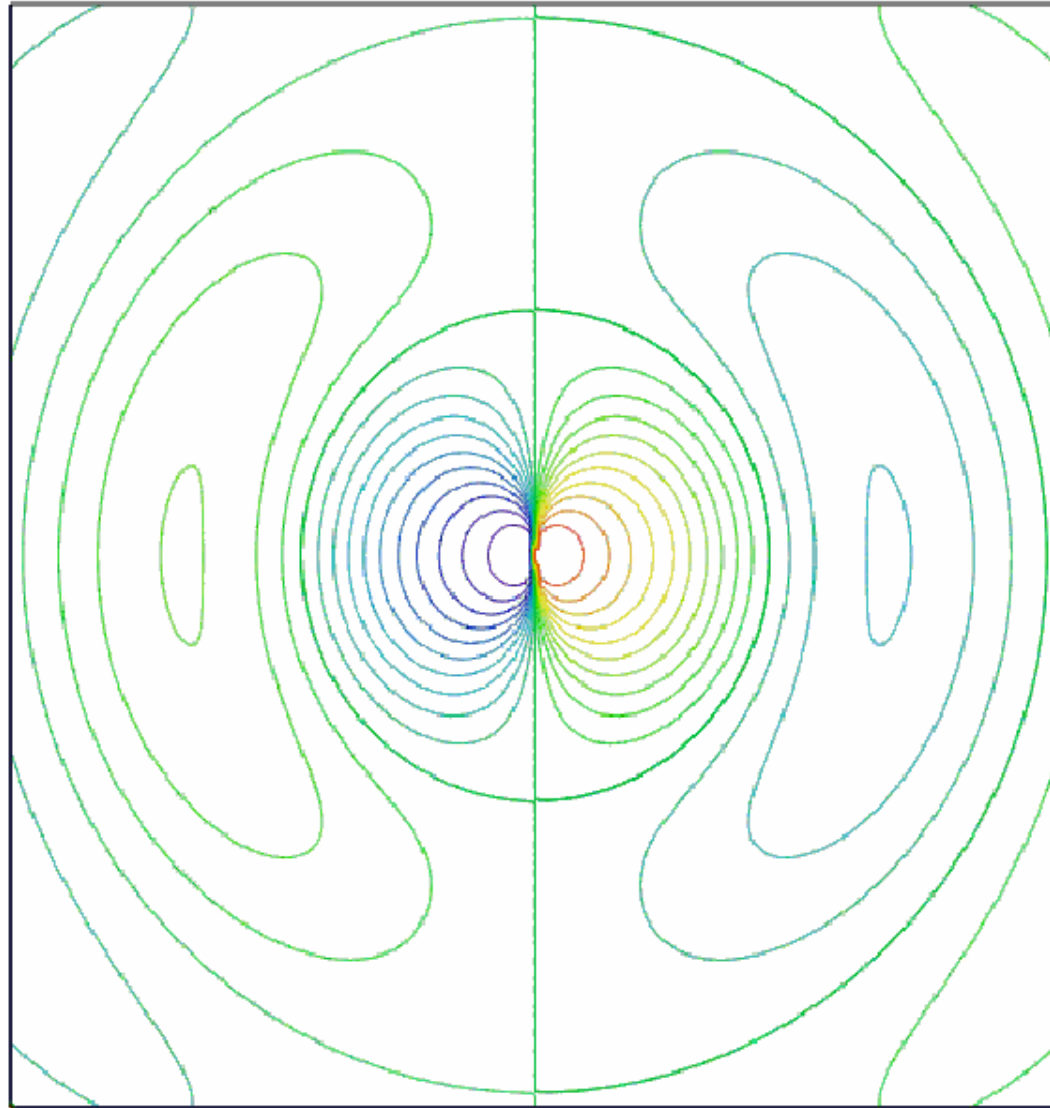
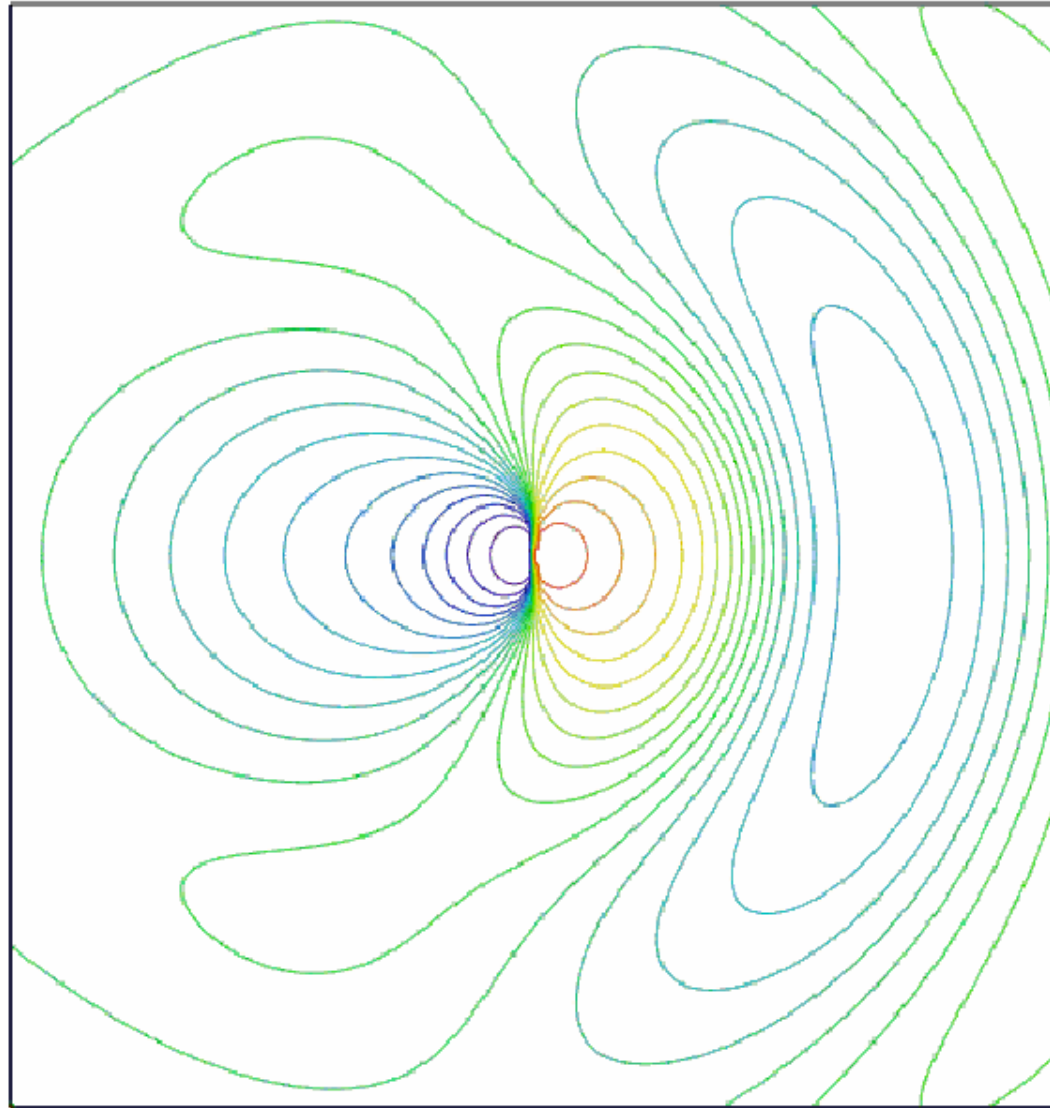


Abb. 6.1/3 e–h

e $t = 29/64 T$ f $t = 30/64 T$ g $t = 31/64 T$ h $t = 32/64 T = T/2$







Allgemeine Quellen

$$\begin{aligned}\vec{E} &= -\mu \frac{\partial}{\partial t} \vec{A}^H - \text{grad} \Phi^H + \text{rot} \vec{A}^E \\ \vec{H} &= \text{rot} \vec{A}^H + \epsilon \frac{\partial}{\partial t} \vec{A}^E + \text{grad} \Phi^E\end{aligned}$$

Fernfeld: $\vec{E} \rightarrow jkZ \vec{e}_r \times \vec{e}_r \times \vec{A}^H + jk \vec{e}_r \times \vec{A}^E$
 $\vec{H} \rightarrow Z^{-1} \vec{e}_r \times \vec{E}$

Fernfeldnäherung: $\|\vec{r} - \vec{r}'\| = \sqrt{r^2 - 2\vec{r} \cdot \vec{r}' + r'^2} \approx r - \vec{e}_r \cdot \vec{r}' \quad r > \frac{2D^2}{\lambda}$

$$\vec{A}^H(\vec{r}) \rightarrow \frac{1}{4\pi} \frac{e^{jkr}}{r} \int_V \vec{J}(\vec{r}') e^{jk\vec{e}_r \cdot \vec{r}'} dV'$$

$$\vec{A}^E(\vec{r}) \rightarrow -\frac{1}{4\pi} \frac{e^{jkr}}{r} \int_V \vec{J}_m(\vec{r}') e^{jk\vec{e}_r \cdot \vec{r}'} dV'$$

3D Fourier Integral

$$\vec{A}^H(\vec{r}) \rightarrow \frac{1}{4\pi} \frac{e^{jkr}}{r} \int_V \vec{J}(\vec{r}') e^{jk\vec{e}_r \cdot \vec{r}'} dV' = \frac{1}{4\pi} \frac{e^{jkr}}{r} \int_V \vec{J}(\vec{r}') e^{j(k_x x' + k_y y' + k_z z')} dV'$$

$$k\vec{e}_r \cdot \vec{r}' = k_x x' + k_y y' + k_z z'$$

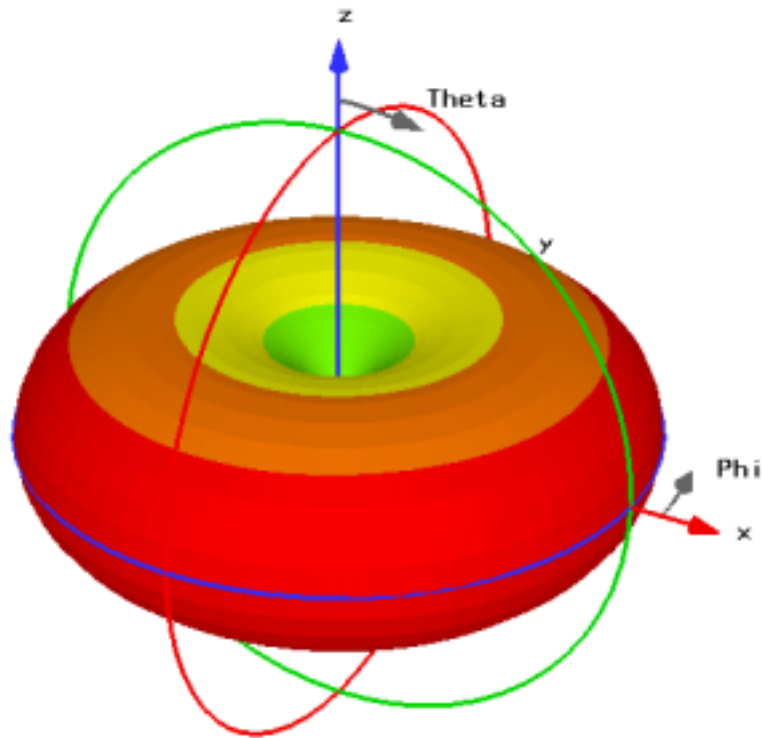
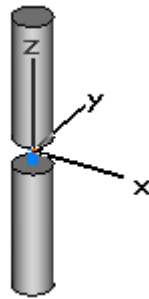
$$F(j\omega) = \int f(t) e^{-j\omega t} dt$$

$$\vec{F}_1(k_x, y', z') = \int \vec{J}(x', y', z') e^{jk_x x'} dx'$$

$$\vec{F}_2(k_x, k_y, z') = \int \vec{F}_1(k_x, y', z') e^{jk_y y'} dy'$$

$$\vec{F}_3(k_x, k_y, k_z) = \int \vec{F}_2(k_x, k_y, z') e^{jk_z z'} dz'$$

$$\int_V \vec{J}(\vec{r}') e^{jk\vec{e}_r \cdot \vec{r}'} dV' = \vec{F}_3(k_x, k_y, k_z)$$



Frequency = 29.9792
 Main lobe magnitude = 1.5
 Main lobe direction = 90.0 deg.
 Angular width [3 dB] = 89.4 deg.

