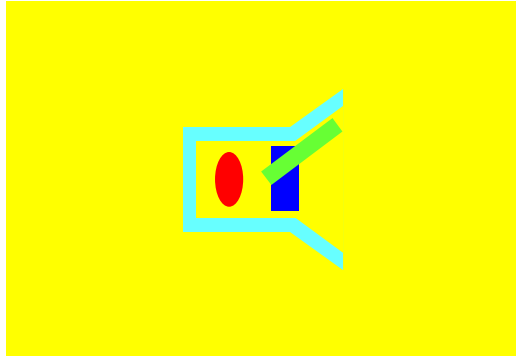


Äquivalente Quellen

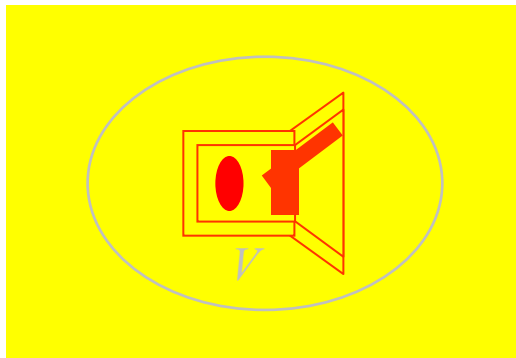


Quellen

Materie: permittiv $\epsilon(\mathbf{r})$
 permeabel $\mu(\mathbf{r})$
 leitfähig $\kappa(\mathbf{r})$

Felder: $\vec{E}(\vec{r})$

$\vec{H}(\vec{r})$

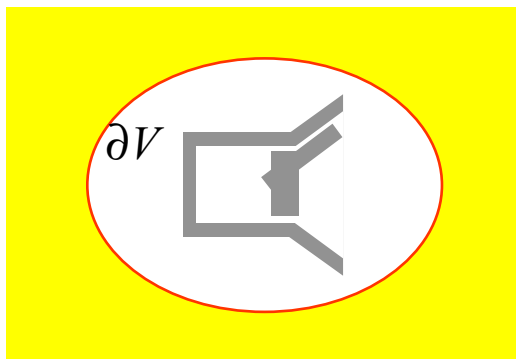


äquivalente Quellen I:
 Volumen Quellen (in V)

Materie: homogen ϵ_0, μ_0

Felder: $\vec{E}(\vec{r})$

$\vec{H}(\vec{r})$



äquivalente Quellen II:
 Flächen Quellen (auf ∂V)

Materie: homogen ϵ_0, μ_0

Felder:

$$\vec{E}^{(0)}(\vec{r}) = \begin{cases} \vec{0} & \text{falls } \vec{r} \in V \\ \vec{E}(\vec{r}) & \text{falls } \vec{r} \notin V \end{cases}$$

$$\vec{H}^{(0)}(\vec{r}) = \begin{cases} \vec{0} & \text{falls } \vec{r} \in V \\ \vec{H}(\vec{r}) & \text{falls } \vec{r} \notin V \end{cases}$$

Felder und Quellen

$$\rho = \varepsilon \operatorname{div} \vec{E}$$

$$\vec{J} = \operatorname{rot} \vec{H} - \varepsilon \frac{\partial}{\partial t} \vec{E}$$

$$\rho_m = \mu \operatorname{div} \vec{H}$$

$$\vec{J}_m = -\operatorname{rot} \vec{E} - \mu \frac{\partial}{\partial t} \vec{H}$$

$$\vec{E} = -\mu \frac{\partial}{\partial t} \vec{A}^H - \operatorname{grad} \Phi^H$$

$$\vec{H} = \operatorname{rot} \vec{A}^H$$

$$+ \operatorname{rot} \vec{A}^E$$

$$+ \varepsilon \frac{\partial}{\partial t} \vec{A}^E + \operatorname{grad} \Phi^E$$

$$\Phi^H(\vec{r}, t) = \frac{1}{4\pi\varepsilon} \int_V \frac{\rho(\vec{r}', t')}{\|\vec{r} - \vec{r}'\|} dV'$$

$$\vec{A}^H(\vec{r}, t) = \frac{1}{4\pi} \int_V \frac{\vec{J}(\vec{r}', t')}{\|\vec{r} - \vec{r}'\|} dV'$$

$$\Phi^E(\vec{r}, t) = -\frac{1}{4\pi\mu} \int_V \frac{\rho_m(\vec{r}', t')}{\|\vec{r} - \vec{r}'\|} dV'$$

$$\vec{A}^E(\vec{r}, t) = -\frac{1}{4\pi} \int_V \frac{\vec{J}_m(\vec{r}', t')}{\|\vec{r} - \vec{r}'\|} dV'$$

Fernfeldnäherung

$$r \gg \frac{2D^2}{\lambda}$$

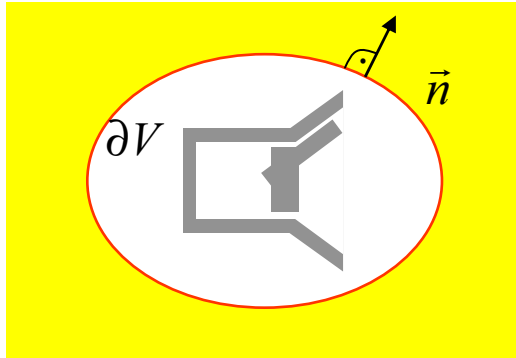
$$kr \gg 1$$

$$\vec{A}^H(\vec{r}) \rightarrow \frac{1}{4\pi} \frac{e^{jkr}}{r} \int_V \vec{J}(\vec{r}') e^{jk\vec{e}_r \cdot \vec{r}'} dV'$$

$$\vec{A}^E(\vec{r}) \rightarrow -\frac{1}{4\pi} \frac{e^{jkr}}{r} \int_V \vec{J}_m(\vec{r}') e^{jk\vec{e}_r \cdot \vec{r}'} dV'$$

$$\vec{E} \rightarrow jkZ \vec{e}_r \times \vec{e}_r \times \vec{A}^H + jk \vec{e}_r \times \vec{A}^E$$

$$\vec{H} \rightarrow Z^{-1} \vec{e}_r \times \vec{E}$$



äquivalente Quellen II:
Flächenquellen (auf ∂V)

$$\vec{J} = \vec{n} \times \vec{H} \quad \rho = \dots$$

$$\vec{J}_m = -\vec{n} \times \vec{E} \quad \rho_m = \dots$$

Materie: homogen ϵ_0, μ_0

Felder:

$$\vec{E}^{(0)}(\vec{r}) = \begin{cases} \vec{0} & \text{falls } \vec{r} \in V \\ \vec{E}(\vec{r}) & \text{falls } \vec{r} \notin V \end{cases}$$

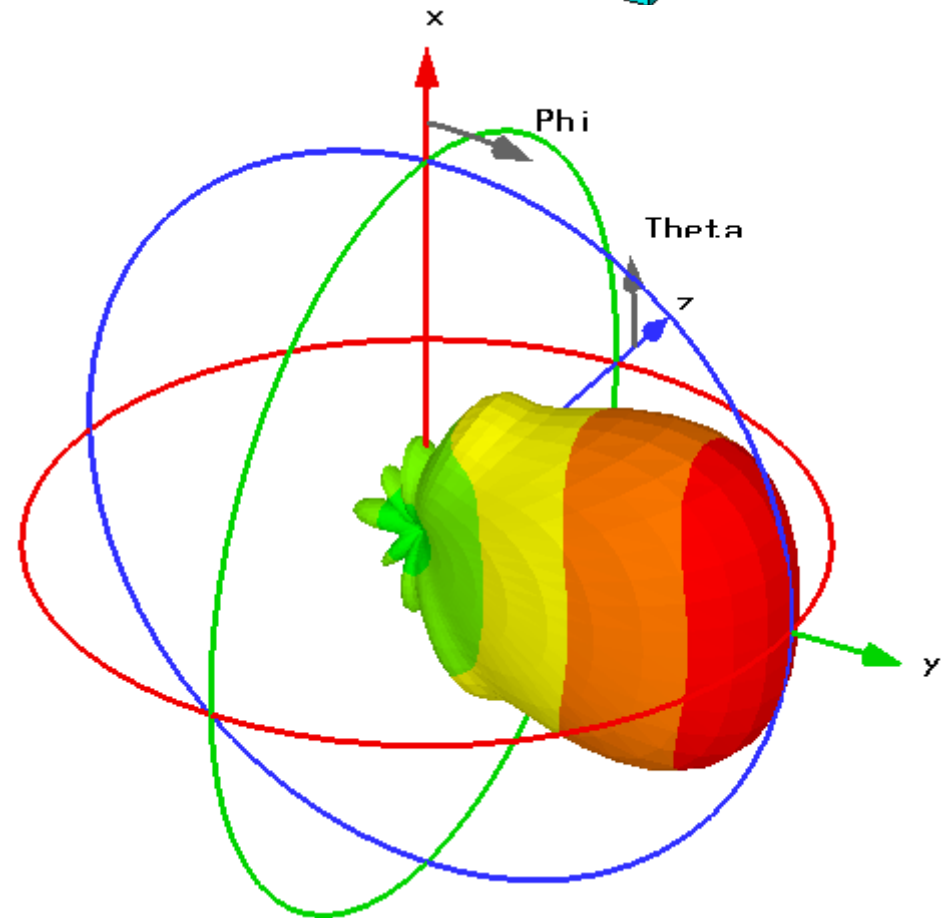
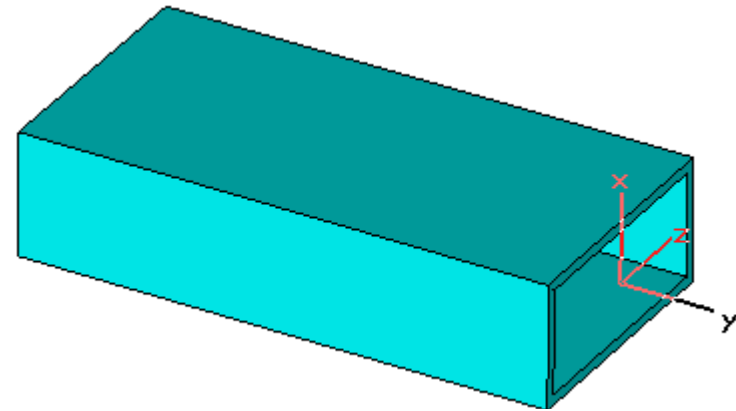
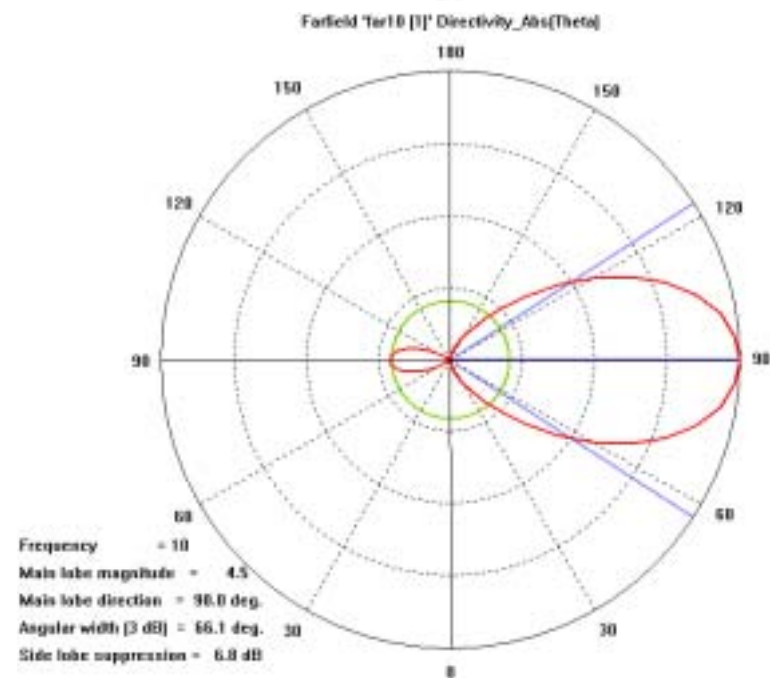
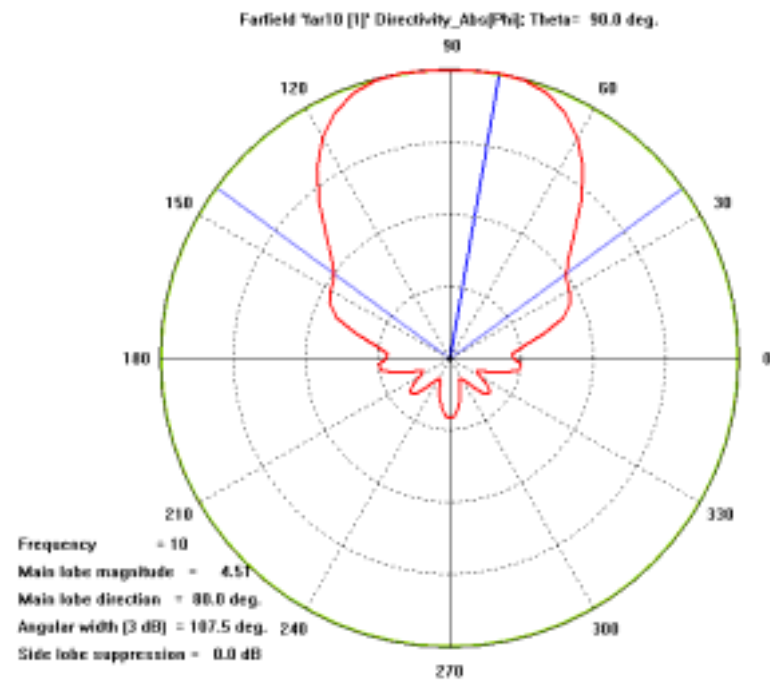
$$\vec{H}^{(0)}(\vec{r}) = \begin{cases} \vec{0} & \text{falls } \vec{r} \in V \\ \vec{H}(\vec{r}) & \text{falls } \vec{r} \notin V \end{cases}$$

$$\vec{A}^H(\vec{r}) \rightarrow \frac{1}{4\pi} \frac{e^{jkr}}{r} \int_{\partial V} (\vec{n} \times \vec{H})_{\vec{r}'} e^{jk\vec{e}_r \cdot \vec{r}'} dA'$$

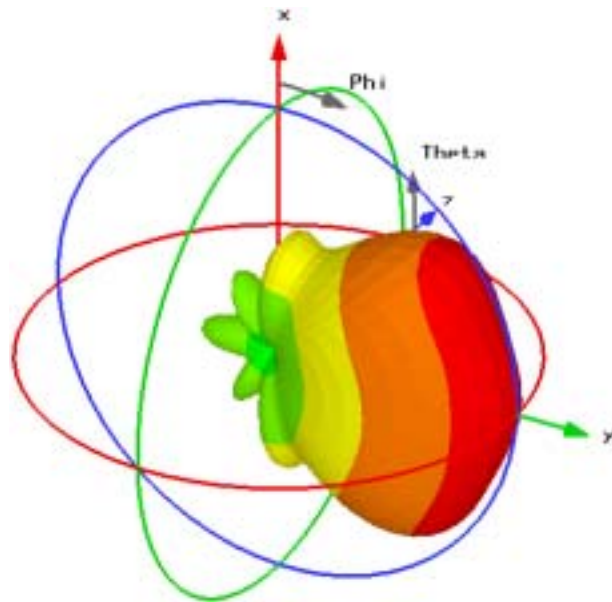
$$\vec{A}^E(\vec{r}) \rightarrow \frac{1}{4\pi} \frac{e^{jkr}}{r} \int_{\partial V} (\vec{n} \times \vec{E})_{\vec{r}'} e^{jk\vec{e}_r \cdot \vec{r}'} dA'$$

$$\vec{E} \rightarrow jkZ \vec{e}_r \times \vec{e}_r \times \vec{A}^H + jk \vec{e}_r \times \vec{A}^E$$

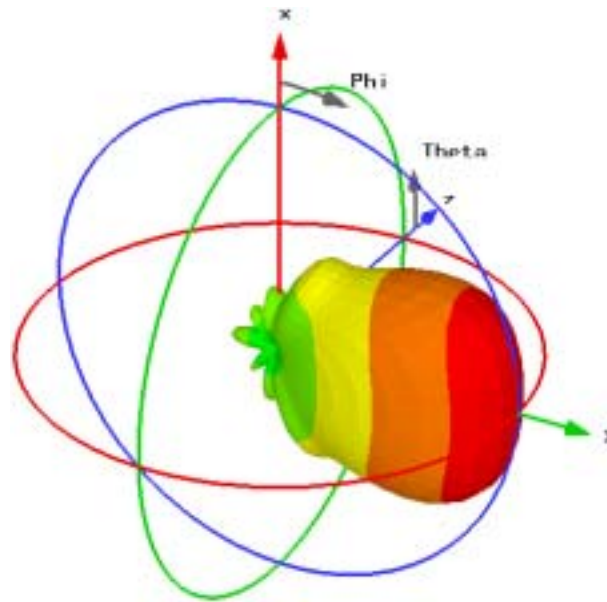
$$\vec{H} \rightarrow Z^{-1} \vec{e}_r \times \vec{E}$$



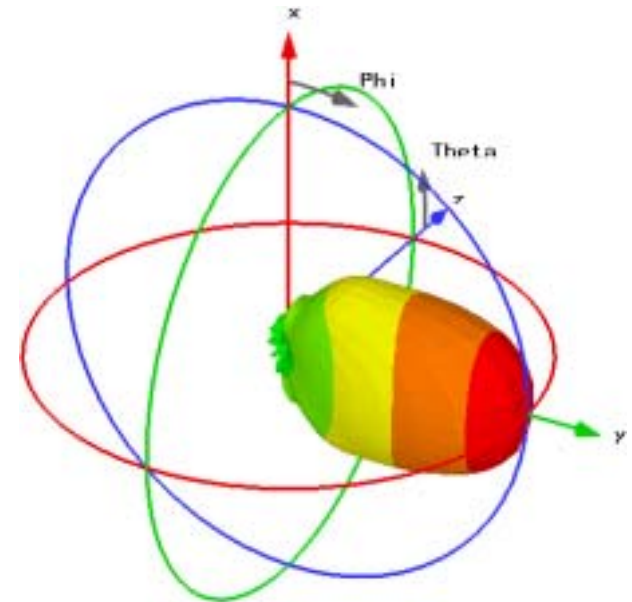
8 GHz



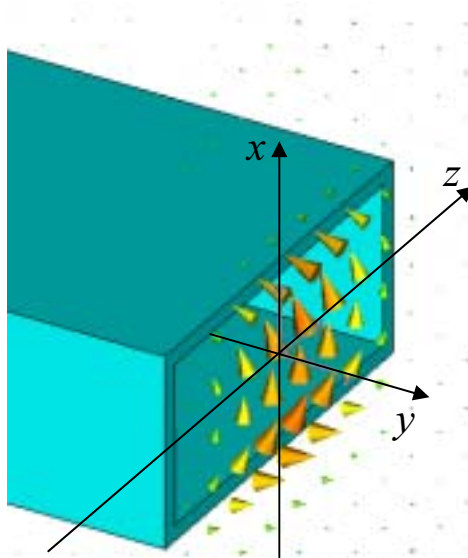
10 GHz



12 GHz



Beispiel



„geschätztes“ Aperturfeld

$$\vec{E}(x,0,z) = \vec{e}_x C \cos\left(\frac{\pi}{a} z\right) \begin{cases} 1 & \text{falls } |z| < a/2, |y| < b/2 \\ 0 & \text{sonst} \end{cases}$$

$$\vec{J}_m = -\vec{e}_y \times \vec{E} = \vec{e}_z C \cos\left(\frac{\pi}{a} z\right) \begin{cases} 1 & \text{falls...} \\ 0 & \text{sonst} \end{cases}$$

$$\vec{A}^E \rightarrow \frac{-1}{4\pi} \frac{e^{-jkr}}{r} \vec{e}_z C \iint_{\substack{|z'| < a/2 \\ |y'| < b/2}} \cos\left(\frac{\pi}{a} z'\right) \exp(jk\vec{e}_r \cdot \vec{r}') dz' dx'$$

$$k\vec{e}_r \cdot \vec{r}' = x' \sin \vartheta \cos \kappa + z' \cos \vartheta$$

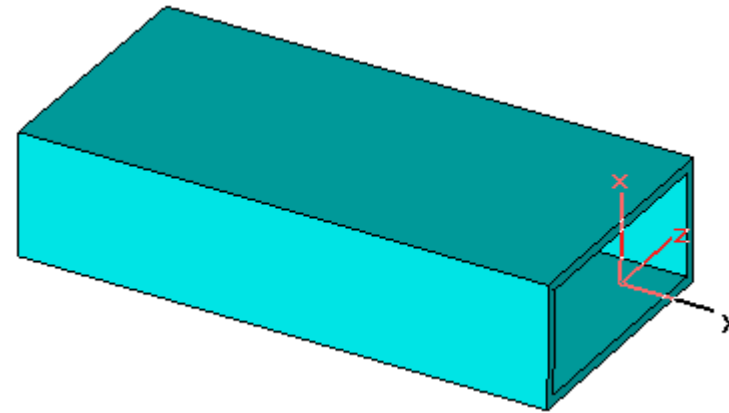
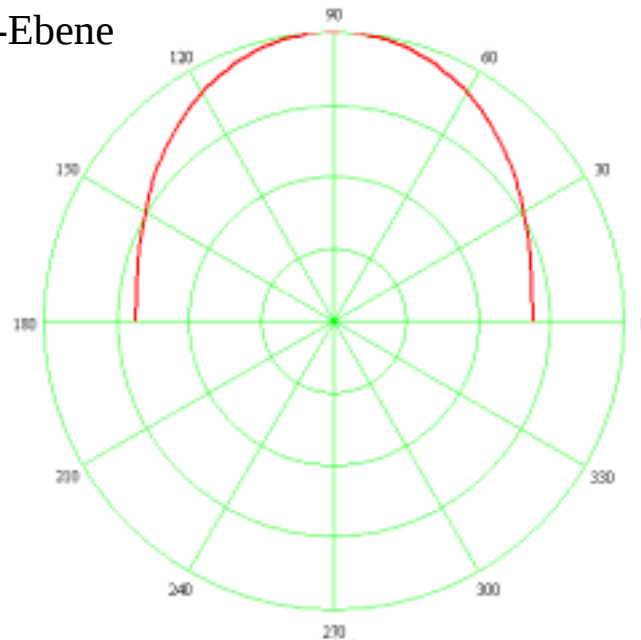
$$\text{z.B. } \varphi = \pi/2: k\vec{e}_r \cdot \vec{r}' = z' \cos \vartheta$$

$$\vec{A}^E(r, \vartheta, \varphi = \pi/2) \rightarrow \frac{-1}{4\pi} \frac{e^{-jkr}}{r} \vec{e}_z C \frac{\cos\left(\frac{ka}{2} \cos \vartheta\right)}{1 - \left(\frac{ka}{\pi}\right)^2 \cos^2 \vartheta}$$

$$\vec{E}^E(r, \vartheta, \varphi = \pi/2) \rightarrow \tilde{C} \frac{e^{-jkr}}{r} \frac{\vec{e}_r \times \vec{e}_z}{-\sin \vartheta \vec{e}_\varphi} \frac{\cos\left(\frac{ka}{2} \cos \vartheta\right)}{1 - \left(\frac{ka}{\pi}\right)^2 \cos^2 \vartheta}$$

$$C(r, \vartheta, \varphi = \pi/2) \propto \left(\frac{\cos\left(\frac{ka}{2} \cos \vartheta\right) \sin \vartheta}{1 - \left(\frac{ka}{\pi}\right)^2 \cos^2 \vartheta} \right)^2$$

y-x-Ebene



Näherung für Aperturfeld

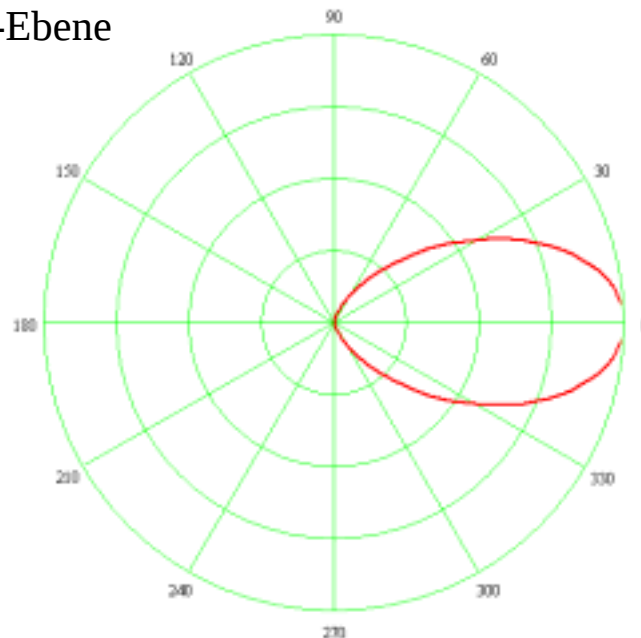
$$E_z \propto \cos\left(\frac{\pi}{a} z\right)$$

$$a = 22.5 \text{ mm}$$

$$b = 10 \text{ mm}$$

$$f = 10 \text{ GHz}$$

y-z-Ebene



Beispiel: Finite Differenzen

$$\begin{aligned} f''(x) &= y(x) & i &= 0, 1, \dots, I \\ f(a) &= \alpha & \Delta x &= (b-a)/I \\ f(b) &= \beta & x_i &= a + i\Delta x \end{aligned}$$

$$f_0 = \alpha$$

$$f_I = \beta$$

$$\frac{1}{\Delta x^2} \begin{bmatrix} -2 & 1 & & & & \\ & 1 & -2 & 1 & & \\ & & 1 & -2 & 1 & \\ & & & \ddots & \ddots & \ddots \\ & & & & \ddots & \ddots & 1 \\ & & & & & 1 & -2 & 1 \\ & & & & & & 1 & -2 & 1 \\ & & & & & & & 1 & -2 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ \vdots \\ \vdots \\ f_{I-3} \\ f_{I-2} \\ f_{I-1} \end{bmatrix} = \begin{bmatrix} y(x_1) - f_0/\Delta x^2 \\ y(x_2) \\ y(x_3) \\ \vdots \\ \vdots \\ y(x_{I-3}) \\ y(x_{I-2}) \\ y(x_{I-1}) - f_I/\Delta x^2 \end{bmatrix}$$