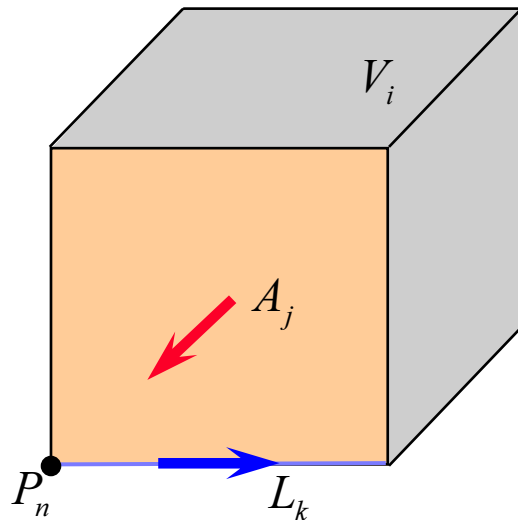


Gitter I



Elementarvolumina

$$V = \{V_1, V_2, \dots, V_{n_V}\}$$

orientierte Elementarflächen

$$A = \{A_1, A_2, \dots, A_{n_A}\}$$

orientierte Elementarstrecken

$$L = \{L_1, L_2, \dots, L_{n_L}\}$$

Punkte

$$P = \{P_1, P_2, \dots, P_{n_P}\}$$

Elementarvolumina: nicht notwendig Quader

Elementarflächen: nicht notwendig eben

Elementarstrecken: nicht notwendig gerade

Zustandsgrößen I

magnetischer Flüsse (durch Elementarflächen)

$$b_j = \iint_{A_j} \vec{B} \cdot d\vec{A}$$

elektrische Spannungen (über Elementarstrecken)

$$e_k = \int_{L_k} \vec{E} \cdot d\vec{s}$$

elektrische Potentiale (der Gitterpunkte)
(z.B. in Elektrostatik)

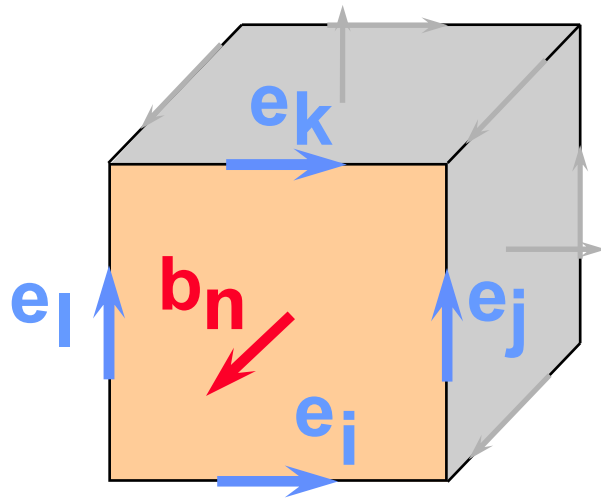
$$\Phi_n = \Phi(P_n)$$

Diskretisierung von „rot E“

$$\oint_{\partial A} \vec{E} \cdot d\vec{s} = -\frac{\partial}{\partial t} \iint_A \vec{B} \cdot d\vec{A}$$

 $\hat{=}$

$$C e = -\frac{\partial}{\partial t} b$$



$$e_i + e_j - e_k - e_l = -\frac{\partial}{\partial t} b_n \quad \rightarrow$$

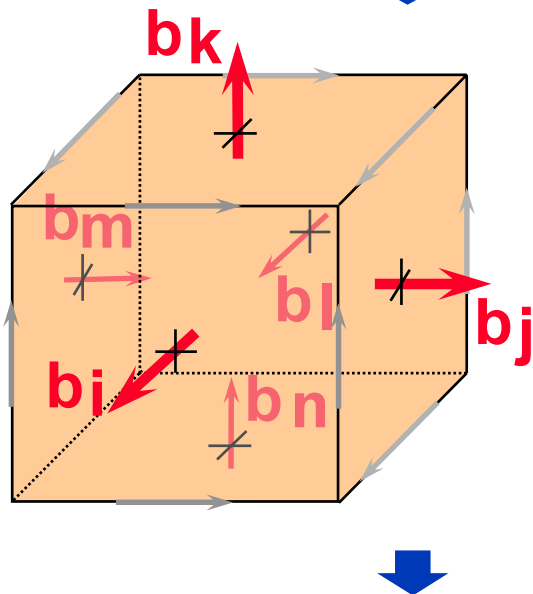
$$\underbrace{\begin{pmatrix} \cdot & \cdot & \cdot \\ 1 & \cdot & 1 & \cdot & -1 & \cdot & -1 \\ \cdot & \cdot & \cdot \end{pmatrix}}_C \underbrace{\begin{pmatrix} e_i \\ \cdot \\ e_j \\ \cdot \\ e_k \\ \cdot \\ e_l \end{pmatrix}}_e = -\frac{\partial}{\partial t} \underbrace{\begin{pmatrix} \cdot \\ b_n \\ \cdot \end{pmatrix}}_b$$

Diskretisierung von „div B“

$$\oiint_{\partial V} \vec{B} \cdot d\vec{A} = 0$$

$\hat{=}$

$$S b = 0$$



$$b_i + b_j + b_k - b_l - b_m - b_n = 0 \quad \Rightarrow$$

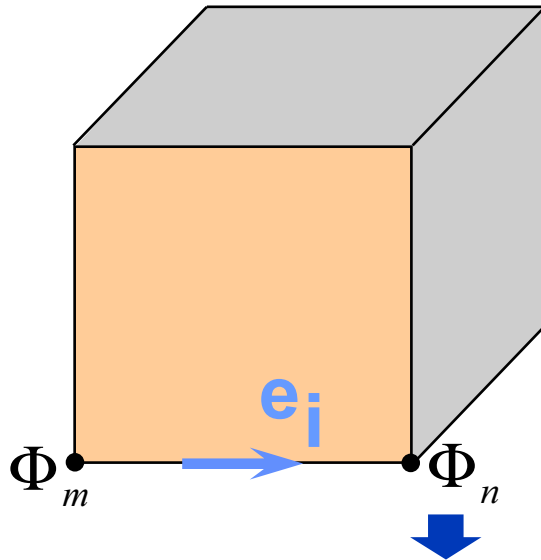
$$\underbrace{\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & 1 & 1 & 1 & -1 & -1 & -1 & \cdot \\ \cdot & \cdot \end{pmatrix}}_S \underbrace{\begin{pmatrix} \cdot \\ b_i \\ b_j \\ b_k \\ b_l \\ b_m \\ b_n \\ \cdot \end{pmatrix}}_b = 0$$

Diskretisierung von „grad Φ “ z.B. Elektrostatik

$$\int_{P_1 \rightarrow P_2} E \cdot d\vec{s} = \Phi_1 - \Phi_2$$

 $\hat{=}$

$$e = -G\Phi$$



$$e_i = -(\Phi_n - \Phi_m)$$



$$\underbrace{\begin{pmatrix} \cdot \\ e_i \\ \cdot \end{pmatrix}}_e = - \begin{pmatrix} \cdot & \cdot \\ -1 & 1 \\ \cdot & \cdot \end{pmatrix} \underbrace{\begin{pmatrix} \cdot \\ \Phi_m \\ \cdot \\ \Phi_n \\ \cdot \end{pmatrix}}_{\Phi}$$

Operatoren I

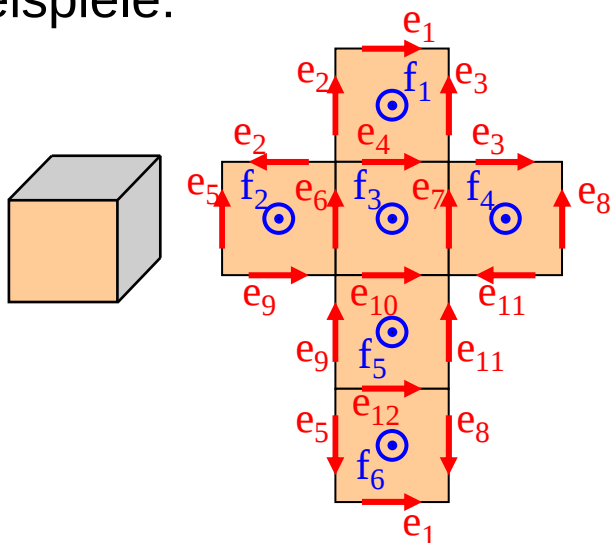
$$\text{div rot} \equiv 0$$

$$SC = 0$$

$$\text{rot grad} \equiv 0$$

$$CG = 0$$

Beispiele:



$$0 = \underbrace{S(Ce)}_f$$

$$f_1 = -e_1 - e_2 + e_3 + e_4$$

$$f_2 = +e_2 - e_5 + e_6 + e_9$$

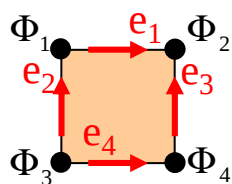
$$f_3 = -e_4 - e_6 + e_7 + e_{10}$$

$$f_4 = -e_3 - e_7 + e_8 - e_{11}$$

$$f_5 = -e_9 - e_{10} + e_{11} + e_{12}$$

$$f_6 = +e_1 + e_5 - e_8 - e_{12}$$

$$f_1 + f_2 + f_3 + f_4 + f_5 + f_6 = \dots = 0$$



$$0 = \underbrace{C(G\Phi)}_{-e}$$

$$e_1 = \Phi_1 - \Phi_2$$

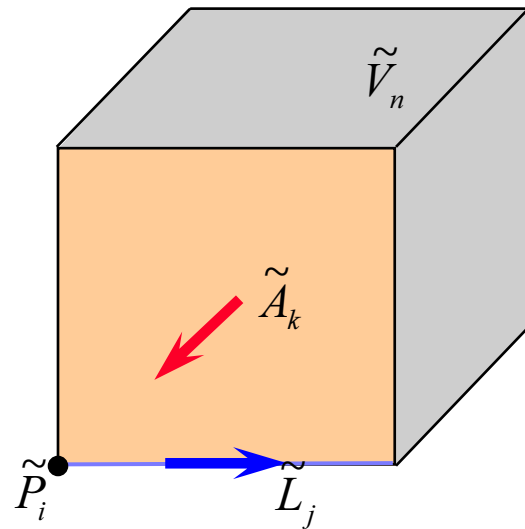
$$e_2 = \Phi_3 - \Phi_1$$

$$e_3 = \Phi_4 - \Phi_2$$

$$e_4 = \Phi_3 - \Phi_4$$

$$-e_1 - e_2 + e_3 + e_4 = \dots = 0$$

Das „andere“ Gitter (Gitter II)



Elementarvolumina

orientierte Elementarflächen

orientierte Elementarstrecken

Punkte

$$\begin{aligned}\tilde{V} &= \{\tilde{V}_1, \tilde{V}_2, \dots, \tilde{V}_{n_{\tilde{V}}}\} \\ \tilde{A} &= \{\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_{n_{\tilde{A}}}\} \\ \tilde{L} &= \{\tilde{L}_1, \tilde{L}_2, \dots, \tilde{L}_{n_{\tilde{L}}}\} \\ \tilde{P} &= \{\tilde{P}_1, \tilde{P}_2, \dots, \tilde{P}_{n_{\tilde{P}}}\}\end{aligned}$$

Elementarvolumina: nicht notwendig Quader

Elementarflächen: nicht notwendig eben

Elementarstrecken: nicht notwendig gerade

Zustandsgrößen II

elektrische Ladungen (in Elementarvolumina)

$$q_n = \iiint_{\tilde{V}_n} \rho dV$$

elektrische Flüsse und Ströme (durch Elementarflächen)

$$d_k = \iint_{A_k} \vec{D} \cdot d\vec{A} \quad i_k = \iint_{A_k} \vec{J} \cdot d\vec{A}$$

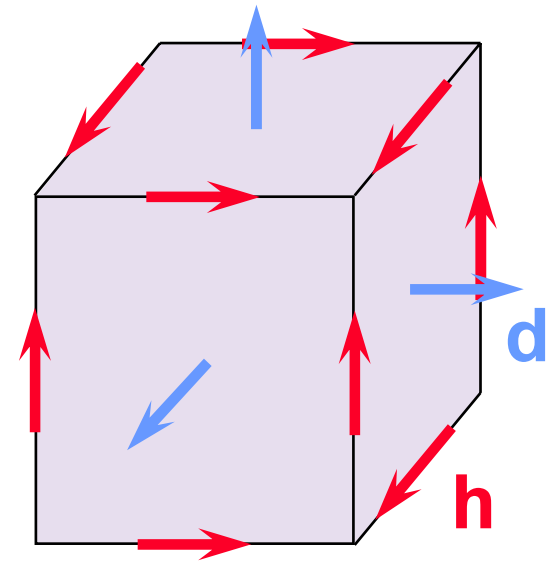
magnetische Spannungen (über Elementarstrecken)

$$h_j = \int_{\tilde{L}_j} \vec{H} \cdot d\vec{s}$$

Diskretisierung von „rot H“ und „div D“ auf dem anderen Gitter ($\tilde{G}$)

$$\oint_{\partial \tilde{A}} \vec{H} \cdot d\vec{s} = \iint_{\tilde{A}} \left(\frac{\partial \vec{D}}{\partial t} + \vec{J} \right) \cdot d\vec{A} \quad \Rightarrow \quad \tilde{C} h = \frac{\partial}{\partial t} d + j$$

$$\iint_{\partial \tilde{V}} \vec{D} \cdot d\vec{A} = \iiint_{\tilde{V}} \rho dV \quad \Rightarrow \quad \tilde{S} d = q$$



Materialgleichungen I

Lokalität und Dualität

$$\vec{D} = \varepsilon \vec{E}$$

$$\vec{B} = \mu \vec{H}$$

$$\vec{J} = \kappa \vec{E} + \vec{J}_q$$

lokale Materialgesetze:

$$\vec{D}(\vec{r}) = \varepsilon(\vec{r}) \vec{E}(\vec{r})$$

Die diskreten Materialgleichungen stellen eine Beziehung her zwischen **Gitter I** und **Gitter II** Zustandsgrößen. Sie verknüpfen Spannungen und Flüsse.

Wunsch:

$$\boxed{\mathbf{d}} = \mathbf{D}_\varepsilon \boxed{\mathbf{e}}$$

$$\mathbf{D}_{\mu^{-1}} \boxed{\mathbf{b}} = \boxed{\mathbf{h}}$$

$$\boxed{\mathbf{j}} = \mathbf{D}_\kappa \boxed{\mathbf{e}} + \boxed{\mathbf{j}_q}$$

$$\left(d_k = \iint_{A_k} \vec{D} \cdot d\vec{A} \right) \approx const_k \times \left(e_k = \int_{L_k} \vec{E} \cdot d\vec{s} \right)$$

$$const_j \times \left(b_j = \iint_{A_j} \vec{B} \cdot d\vec{A} \right) \approx \left(h_j = \int_{\tilde{L}_j} \vec{H} \cdot d\vec{s} \right)$$

(dann sind die
Materialmatrizen diagonal)

Duale Elemente

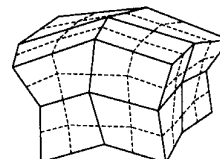
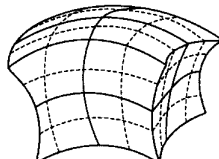
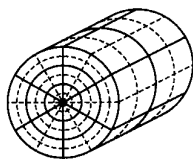
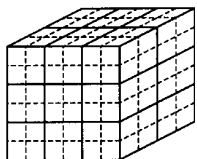
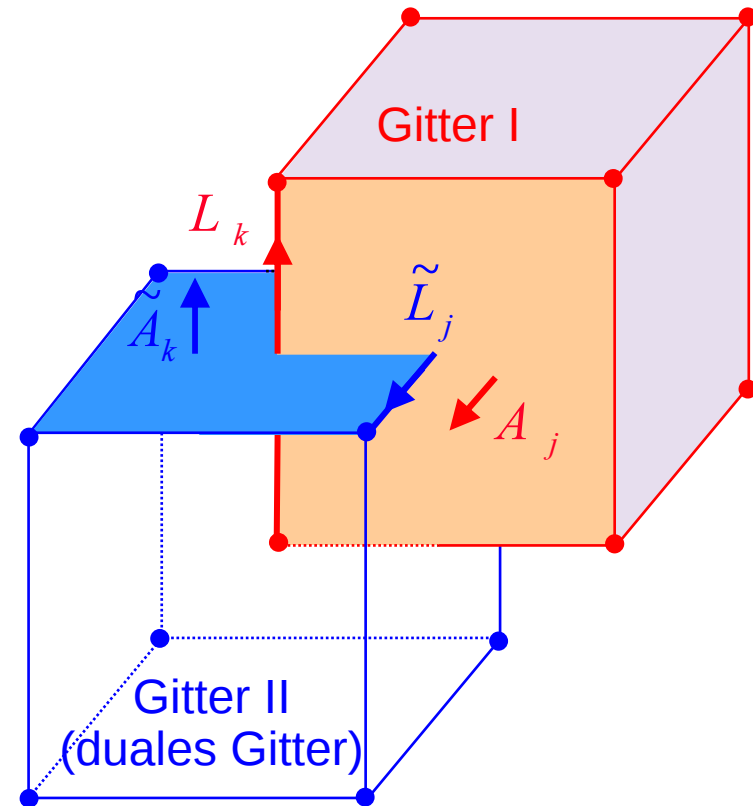
$$\text{const}_j \times \left(b_j = \iint_{A_j} \vec{B} \cdot d\vec{A} \right) \approx \left(h_j = \int_{\tilde{L}_j} \vec{H} \cdot d\vec{s} \right)$$

$$\left(d_k = \iint_{\tilde{A}_k} \vec{D} \cdot d\vec{A} \right) \approx \text{const}_k \times \left(e_k = \int_{L_k} \vec{E} \cdot d\vec{s} \right)$$

$$\begin{array}{l} V = \{V_1, V_2, \dots, V_{n_V}\} \\ A = \{A_1, A_j, \dots, A_{n_A}\} \\ L = \{L_1, L_k, \dots, L_{n_L}\} \\ P = \{P_1, P_2, \dots, P_{n_P}\} \end{array} \quad \longleftrightarrow \quad \begin{array}{l} \tilde{P} = \{\tilde{P}_1, \tilde{P}_2, \dots, \tilde{P}_{n_{\tilde{P}}}\} \\ \tilde{L} = \{\tilde{L}_1, \tilde{L}_j, \dots, \tilde{L}_{n_{\tilde{L}}}\} \\ \tilde{A} = \{\tilde{A}_1, \tilde{A}_k, \dots, \tilde{A}_{n_{\tilde{A}}}\} \\ \tilde{V} = \{\tilde{V}_1, \tilde{V}_2, \dots, \tilde{V}_{n_{\tilde{V}}}\} \end{array}$$



duale Gitter



Operatoren II

Gitter I

Gitter II

$$\operatorname{div} \operatorname{rot} \equiv 0$$

$$SC = 0$$

$$\tilde{S}\tilde{C} = 0$$

$$\operatorname{rot} \operatorname{grad} \equiv 0$$

$$CG = 0$$

$$\tilde{C}\tilde{G} = 0$$

duale Gitter

$$C = \tilde{C}^t$$

$$G = -\tilde{S}^t$$

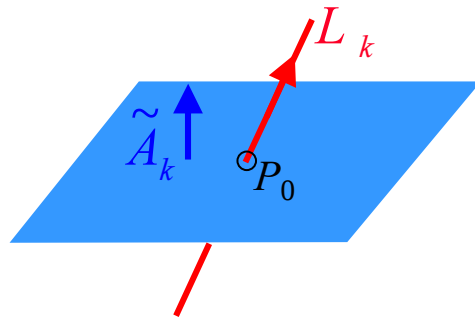
$$S = -\tilde{G}^t$$

Materialgleichungen II

Orthogonalität

Beispiel: homogenes Volumen,
ebene bzw. lineare Elemente

$$\left(d_k = \iint_{A_k} \vec{D} \cdot d\vec{A} \right) \approx (\mathbf{D}_\varepsilon)_k \times \left(e_k = \int_{L_k} \vec{E} \cdot d\vec{s} \right)$$



$$\vec{E}(\vec{r}) = \vec{E}(\vec{r}_0) + O(\delta\vec{r})$$

$$\vec{D}(\vec{r}) = \varepsilon \vec{E}(\vec{r}_0) + O(\delta\vec{r})$$

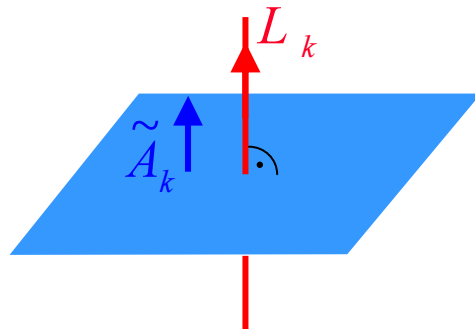
$$\vec{E}(\vec{r}_0) = \vec{E}_0$$

$$d_k = \iint_{A_k} \vec{D} \cdot d\vec{A} = \varepsilon \underbrace{\iint_{A_k} dA}_{\tilde{a}_k} (\vec{E}_0 \cdot \vec{e}_{\tilde{A}}) + O_d$$

$$e_k = \int_{L_k} \vec{E} d\vec{s} = \underbrace{\int_{L_k} ds}_{l_k} (\vec{E}_0 \cdot \vec{e}_s) + O_e$$

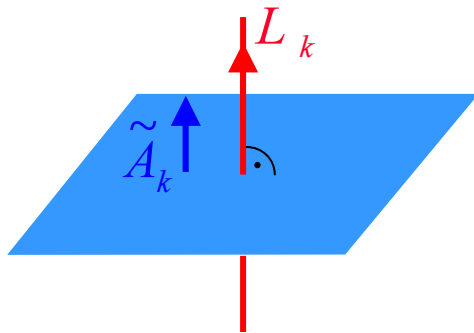
$$(\varepsilon \tilde{a}_k (\vec{E}_0 \cdot \vec{e}_{\tilde{A}}) + O_d) \rightarrow (\mathbf{D}_\varepsilon)_k \times (l_k (\vec{E}_0 \cdot \vec{e}_s) + O_e)$$

Orthogonalität der dualen Elemente: $\vec{e}_{\tilde{A}} = \vec{e}_s = \vec{e}$



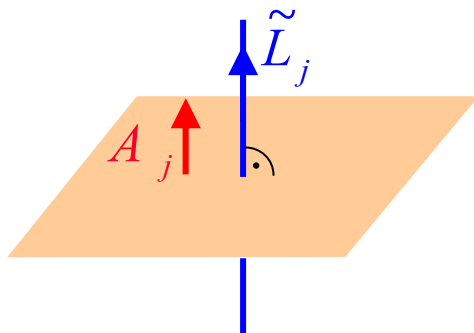
$$(\varepsilon \tilde{a}_k (\vec{E}_0 \cdot \vec{e}) + O_d) \approx \underbrace{\varepsilon \frac{\tilde{a}_k}{l_k}}_{(\mathbf{D}_\varepsilon)_k} \times (l_k (\vec{E}_0 \cdot \vec{e}) + O_e)$$

Beispiel: homogenes Volumen,
ebene bzw. lineare Elemente



$$\left(d_k = \iint_{A_k} \vec{D} \cdot d\vec{A} \right) \approx \underbrace{\varepsilon \frac{\tilde{a}_k}{l_k}}_{(\mathbf{D}_\varepsilon)_k} \times \left(e_k = \int_{L_k} \vec{E} \cdot d\vec{s} \right)$$

$$\left(j_k = \iint_{A_k} \vec{J} \cdot d\vec{A} \right) \approx \underbrace{\kappa \frac{\tilde{a}_k}{l_k}}_{(\mathbf{D}_\kappa)_k} \times \left(e_k = \int_{L_k} \vec{E} \cdot d\vec{s} \right) + \left(j_{q,k} = \iint_{A_k} \vec{J}_q \cdot d\vec{A} \right)$$

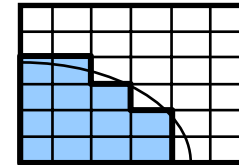
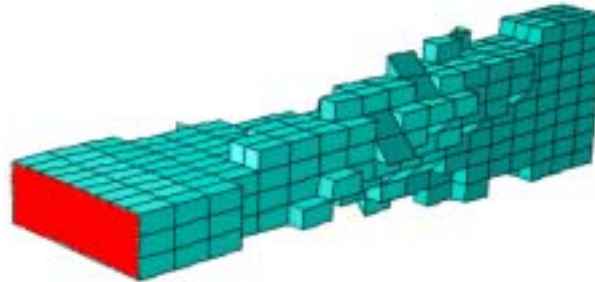


$$\underbrace{\frac{1}{\mu} \frac{\tilde{l}_j}{a_j}}_{(\mathbf{D}_{\mu^{-1}})_j} \times \left(b_j = \iint_{A_j} \vec{B} \cdot d\vec{A} \right) \approx \left(h_j = \int_{\tilde{L}_j} \vec{H} \cdot d\vec{s} \right)$$

Zellfüllungen

ortsabhängige Materialeigenschaften

klassische FI-Methode



Gitterzellen sind jeweils homogen gefüllt

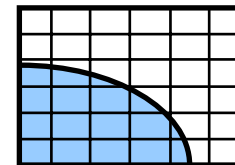
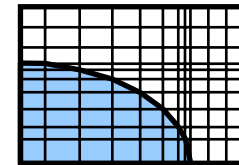
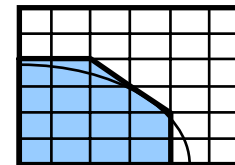
Alternative: die dualen Gitterzellen sind jeweils homogen gefüllt

Erweiterungen

Dreiecksprismen: MAFIA

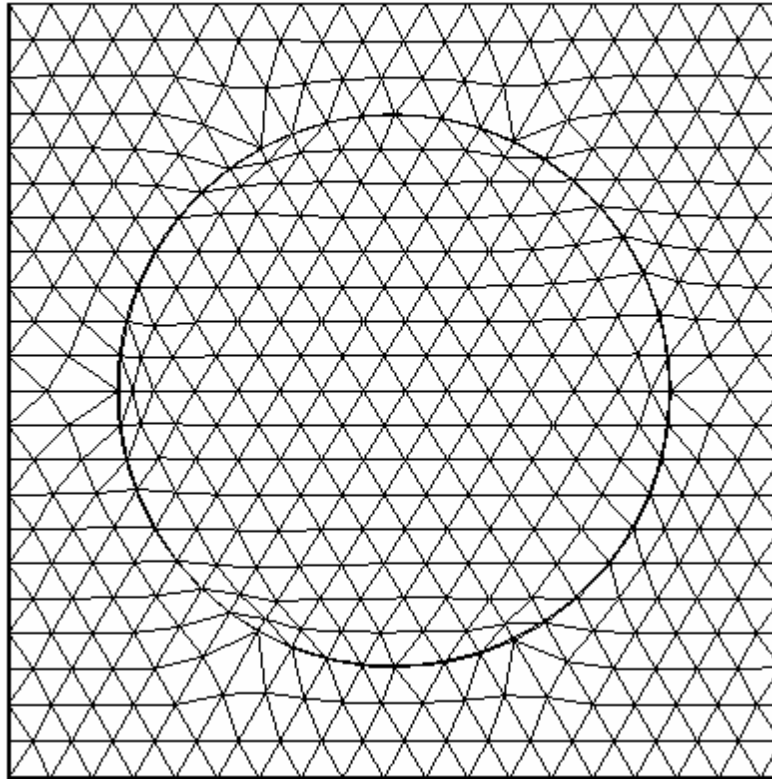
teilweise Zellfüllungen: MWS

...

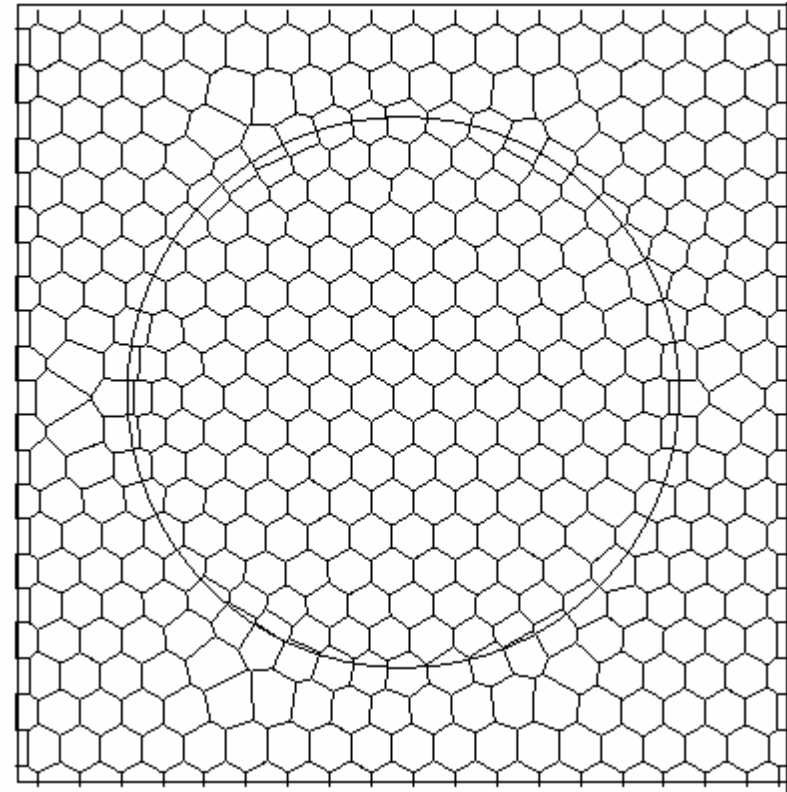


Dreiecksgitter

Gitter



duales Gitter



FI-Methode

$$\begin{aligned}\operatorname{div} \vec{D} &= \rho \\ \operatorname{rot} \vec{E} &= -\frac{\partial}{\partial t} \vec{B} \\ \operatorname{div} \vec{B} &= 0 \\ \operatorname{rot} \vec{H} &= \vec{J} + \frac{\partial}{\partial t} \vec{D}\end{aligned}$$

$$\begin{aligned}\vec{D} &= \varepsilon \vec{E} \\ \vec{B} &= \mu \vec{H} \\ \vec{J} &= \kappa \vec{E} + \vec{J}_q\end{aligned}$$

RB

$$\begin{aligned}\operatorname{div} \operatorname{rot} &\equiv 0 \\ \operatorname{rot} \operatorname{grad} &\equiv 0\end{aligned}$$

$$\begin{aligned}\tilde{\mathbf{S}} \mathbf{d} &= \mathbf{q} \\ \mathbf{C} \mathbf{e} &= -\frac{\partial}{\partial t} \mathbf{b} \\ \mathbf{S} \mathbf{b} &= \mathbf{0} \\ \tilde{\mathbf{C}} \mathbf{h} &= \mathbf{j} + \frac{\partial}{\partial t} \mathbf{d}\end{aligned}$$

$$\begin{aligned}\mathbf{d} &= \mathbf{D}_\varepsilon \mathbf{e} \\ \mathbf{D}_{\mu^{-1}} \mathbf{b} &= \mathbf{h} \\ \mathbf{j} &= \mathbf{D}_\kappa \mathbf{e} + \mathbf{j}_q\end{aligned}$$

$$\begin{aligned}SC &= 0 & \tilde{\mathbf{S}} \tilde{\mathbf{C}} &= 0 & G &= -\tilde{\mathbf{S}}^t \\ CG &= 0 & \tilde{\mathbf{C}} \tilde{\mathbf{G}} &= 0 & S &= -\tilde{\mathbf{G}}^t \\ C &= \tilde{\mathbf{C}}^t\end{aligned}$$

Beispiel: Elektrostatik

$$\begin{aligned}\operatorname{div} \vec{D} &= \rho \\ \operatorname{rot} \vec{E} &= 0 \rightarrow \vec{E} = -\operatorname{grad} \Phi \\ \vec{D} &= \varepsilon \vec{E} \\ \text{RB}\end{aligned}$$



$$\begin{aligned}\operatorname{div}(\varepsilon \operatorname{grad} \Phi) &= -\rho \\ \text{RB}\end{aligned}$$

$$\begin{aligned}\tilde{\mathbf{S}} \mathbf{d} &= \mathbf{q} \\ \mathbf{C} \mathbf{e} = \mathbf{0} &\rightarrow \mathbf{e} = -\mathbf{G} \Phi = \tilde{\mathbf{S}}^t \Phi \\ \mathbf{d} &= \mathbf{D}_\varepsilon \mathbf{e}\end{aligned}$$



$$\tilde{\mathbf{S}} \mathbf{D}_\varepsilon \tilde{\mathbf{S}}^t \Phi = \mathbf{q}$$

$\tilde{\mathbf{S}} \mathbf{D}_\varepsilon \tilde{\mathbf{S}}^t$
ist symmetrisch
und positiv definit

Kontinuitätsgleichung

$$\operatorname{div} \vec{D} = \rho$$

$$\operatorname{rot} \vec{H} = \vec{J} + \frac{\partial}{\partial t} \vec{D}$$

$$0 = \operatorname{div}(\operatorname{rot} \vec{H}) = \operatorname{div} \vec{J} + \frac{\partial}{\partial t} \operatorname{div} \vec{D}$$

$$0 = \operatorname{div} \vec{J} + \frac{\partial \rho}{\partial t}$$

$$\tilde{\mathbf{S}} \mathbf{d} = \mathbf{q}$$

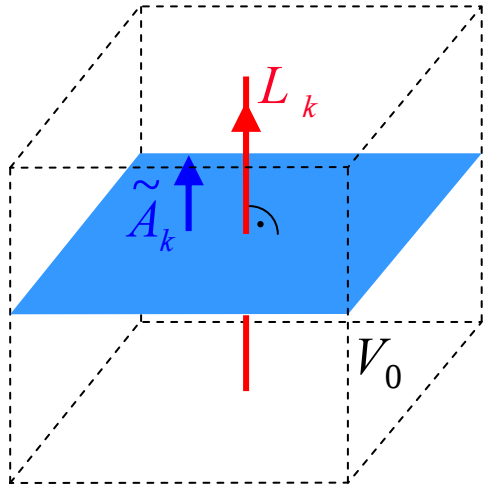
$$\tilde{\mathbf{C}} \mathbf{h} = \mathbf{j} + \frac{\partial}{\partial t} \mathbf{d}$$

$$\mathbf{0} = \tilde{\mathbf{S}} \tilde{\mathbf{C}} \mathbf{h} = \tilde{\mathbf{S}} \mathbf{j} + \frac{\partial}{\partial t} \tilde{\mathbf{S}} \mathbf{d}$$

$$\mathbf{0} = \tilde{\mathbf{S}} \mathbf{j} + \frac{\partial}{\partial t} \mathbf{q}$$

Energie im Gitter

Beispiel: kartesisches Gitter



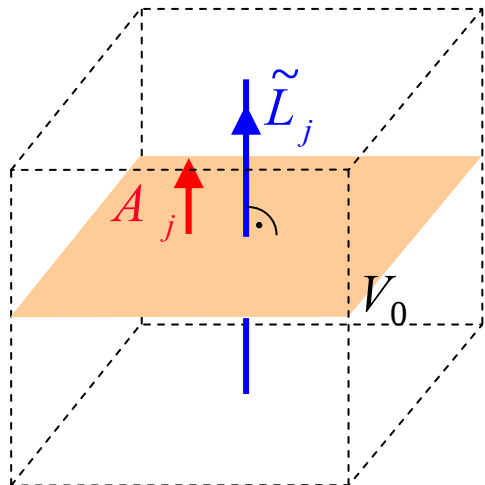
$$e_k d_k = \left(\int_{L_k} \vec{E} \cdot \vec{e}_k ds \right) \left(\int_{\tilde{A}_k} \vec{D} \cdot \vec{e}_k dA \right)$$

$$\approx (\vec{E}_0 \cdot \vec{e}_k) (\vec{D}_0 \cdot \vec{e}_k) \tilde{a}_k$$

$$\approx \int_{V_0} (\vec{E} \cdot \vec{e}_k) (\vec{D} \cdot \vec{e}_k) dV$$

$$\sum_k e_k d_k = \mathbf{e}^t \mathbf{d} \approx \int_V \vec{E} \cdot \vec{D} dV = 2W_e$$

$$\sum_k e_k j_k = \mathbf{e}^t \mathbf{j} \approx \int_V \vec{E} \cdot \vec{J} dV = P_{Joule}$$



$$\sum_j h_j b_j = \mathbf{h}^t \mathbf{b} \approx \int_V \vec{H} \cdot \vec{B} dV = 2W_m$$

gilt
allgemein

Energieerhaltung

$$\begin{aligned}
 \frac{\partial}{\partial t} W_{tot} &= \frac{\partial}{\partial t} (W_e + W_m) \\
 &= \frac{1}{2} \frac{\partial}{\partial t} \int_V (\vec{E} \cdot \vec{D} + \vec{H} \cdot \vec{B}) dV \\
 &= \int_V \left(\vec{E} \cdot \frac{\partial}{\partial t} \vec{D} + \vec{H} \cdot \frac{\partial}{\partial t} \vec{B} \right) dV \\
 &= \int_V (\vec{E} \cdot (\text{rot } \vec{H} - \vec{J}) - \vec{H} \cdot \text{rot } \vec{E}) dV \\
 &= - \int_V (\vec{E} \cdot \vec{J}) dV - \int_V \text{div}(\vec{E} \times \vec{H}) dV
 \end{aligned}$$

$$\frac{\partial}{\partial t} W_{tot} = - \int_V \vec{E} \cdot \vec{J} dV - \underbrace{\int_{\partial V} (\vec{E} \times \vec{H}) \cdot d\vec{A}}_{=0 \text{ für PEC/PMC RB}}$$

0 für PEC/PMC RB

$$\begin{aligned}
 \frac{\partial}{\partial t} W_{tot} &= \frac{\partial}{\partial t} (W_e + W_m) \\
 &= \frac{1}{2} \frac{\partial}{\partial t} (\mathbf{e}^t \mathbf{d} + \mathbf{h}^t \mathbf{b}) \\
 &= \mathbf{e}^t \frac{\partial}{\partial t} \mathbf{d} + \mathbf{h}^t \frac{\partial}{\partial t} \mathbf{b} \\
 &= \mathbf{e}^t (\tilde{\mathbf{C}} \mathbf{h} - \mathbf{j}) - \mathbf{h}^t \mathbf{C} \mathbf{e}
 \end{aligned}$$

$$\frac{\partial}{\partial t} W_{tot} = -\mathbf{e}^t \mathbf{j}$$

Zeitbereich (verlustfrei)

AWA: $\vec{B}(t_0) = \vec{B}_0 \quad \frac{\partial}{\partial t} \vec{B} = \text{rot } \vec{E}$

$\vec{E}(t_0) = \vec{E}_0 \quad \frac{\partial}{\partial t} \vec{E} = \frac{1}{\varepsilon} \text{rot } \vec{H} - \frac{1}{\varepsilon} \vec{J}$

$$\vec{B}^0 = \vec{B}(t_0)$$

$$\vec{E}^{1/2} = \vec{E}(t_0 + \delta t/2)$$

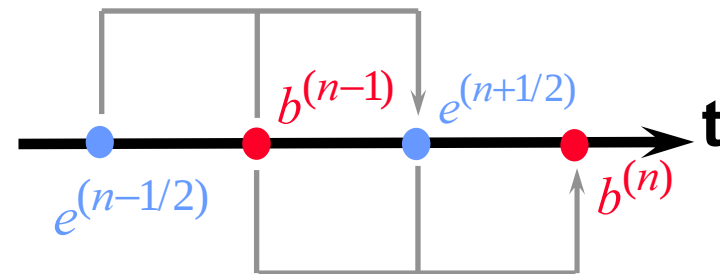
$$\frac{\vec{B}^{n+1} - \vec{B}^n}{\delta t} \approx \text{rot } \vec{E}^{n+1/2}$$

$$\frac{\vec{E}^{n+3/2} - \vec{E}^{n+1/2}}{\delta t} \approx \frac{1}{\varepsilon} \text{rot} \left(\frac{1}{\mu} \vec{B}^{n+1} \right) - \frac{1}{\varepsilon} \vec{J}^{n+1}$$

$$\vec{B}^{n+1} = \vec{B}^n + \delta t \text{rot } \vec{E}^{n+1/2}$$

$$\vec{E}^{n+3/2} = \vec{E}^{n+1/2} + \delta t \left(\frac{1}{\varepsilon} \text{rot} \left(\frac{1}{\mu} \vec{B}^{n+1} \right) - \frac{1}{\varepsilon} \vec{J}^{n+1} \right)$$

$$\frac{\partial f}{\partial t} \approx \frac{f(t + \delta t/2) - f(t - \delta t/2)}{\delta t}$$

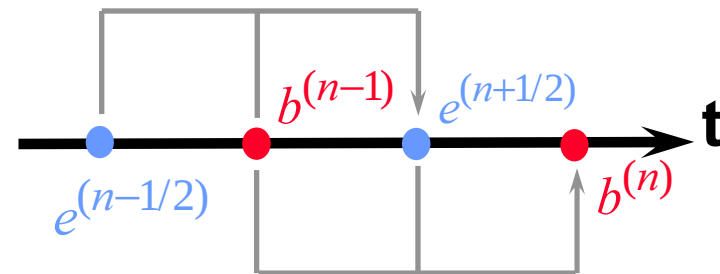


$$\vec{B}^{n+1} = \vec{B}^n + \delta t \operatorname{rot} \vec{E}^{n+1/2}$$

$$\vec{E}^{n+3/2} = \vec{E}^{n+1/2} + \delta t \left(\frac{1}{\varepsilon} \operatorname{rot} \left(\frac{1}{\mu} \vec{B}^{n+1} \right) - \frac{1}{\varepsilon} \vec{J}^{n+1} \right)$$

$$\mathbf{b}^{n+1} = \mathbf{b}^n + \delta t \mathbf{C} \mathbf{e}^{n+1/2}$$

$$\mathbf{e}^{n+3/2} = \mathbf{e}^{n+1/2} + \delta t \left(\mathbf{D}_\varepsilon^{-1} \tilde{\mathbf{C}} \mathbf{D}_{\mu^{-1}} \mathbf{b}^{n+1} - \frac{1}{\varepsilon} \mathbf{j}^{n+1} \right)$$



Verlustfreie Resonatoren (ungedämpfte Eigenmoden)

$$\begin{aligned}\operatorname{rot} \vec{E} &= -j\omega \vec{B} \\ \operatorname{rot} \frac{1}{\mu} \vec{B} &= j\omega \epsilon \vec{E}\end{aligned}$$

RB

$$\operatorname{rot} \left(\frac{1}{\mu} \operatorname{rot} \vec{E} \right) = \omega^2 \epsilon \vec{E}$$

$$\begin{aligned}\mathbf{C} \mathbf{e} &= -j\omega \mathbf{b} \\ \tilde{\mathbf{C}} \mathbf{D}_{\mu^{-1}} \mathbf{b} &= j\omega \mathbf{D}_{\epsilon} \mathbf{e}\end{aligned}$$

$$\tilde{\mathbf{C}} \mathbf{D}_{\mu^{-1}} \mathbf{C} \mathbf{e} = \omega^2 \mathbf{D}_{\epsilon} \mathbf{e}$$



sym. EWP

$$\mathbf{A} \mathbf{x} = \omega^2 \mathbf{x}$$

$\mathbf{A}$ positiv semidefinit

$$\Rightarrow \omega_v \in \Re^{+0}$$

weitere Eigenschaften:

$$\omega_v > 0 \Rightarrow \operatorname{div} \vec{D} = 0$$

$$\begin{aligned} \tilde{\mathbf{C}} \mathbf{D}_{\mu^{-1}} \mathbf{C} \mathbf{e}_v &= \omega_v^2 \mathbf{D}_{\varepsilon} \mathbf{e}_v \\ \underbrace{\tilde{\mathbf{S}} \tilde{\mathbf{C}}}_{\mathbf{0}} \mathbf{D}_{\mu^{-1}} \mathbf{C} \mathbf{e}_v &= \omega_v^2 \underbrace{\tilde{\mathbf{S}} \mathbf{D}_{\varepsilon}}_{\mathbf{d}_v} \mathbf{e}_v \end{aligned}$$

$$\omega_v > 0 \Rightarrow \tilde{\mathbf{S}} \mathbf{d}_v = \mathbf{0}$$

$$W_e = W_m$$

$$\begin{aligned} 2W_e &= \overline{\mathbf{e}^t} \mathbf{d} \\ 2W_m &= \overline{\mathbf{b}^t} \mathbf{h} \\ &= \overline{\left(\frac{1}{-j\omega_v} \mathbf{C} \mathbf{e} \right)^t} \left(\frac{1}{-j\omega_v} \mathbf{D}_{\mu^{-1}} \mathbf{C} \mathbf{e} \right) \\ &= \frac{1}{\omega_v^2} \overline{\mathbf{e}^t} \underbrace{\tilde{\mathbf{C}} \mathbf{D}_{\mu^{-1}} \mathbf{C}}_{\omega_v^2 \mathbf{d}} \mathbf{e} = 2W_e \end{aligned}$$