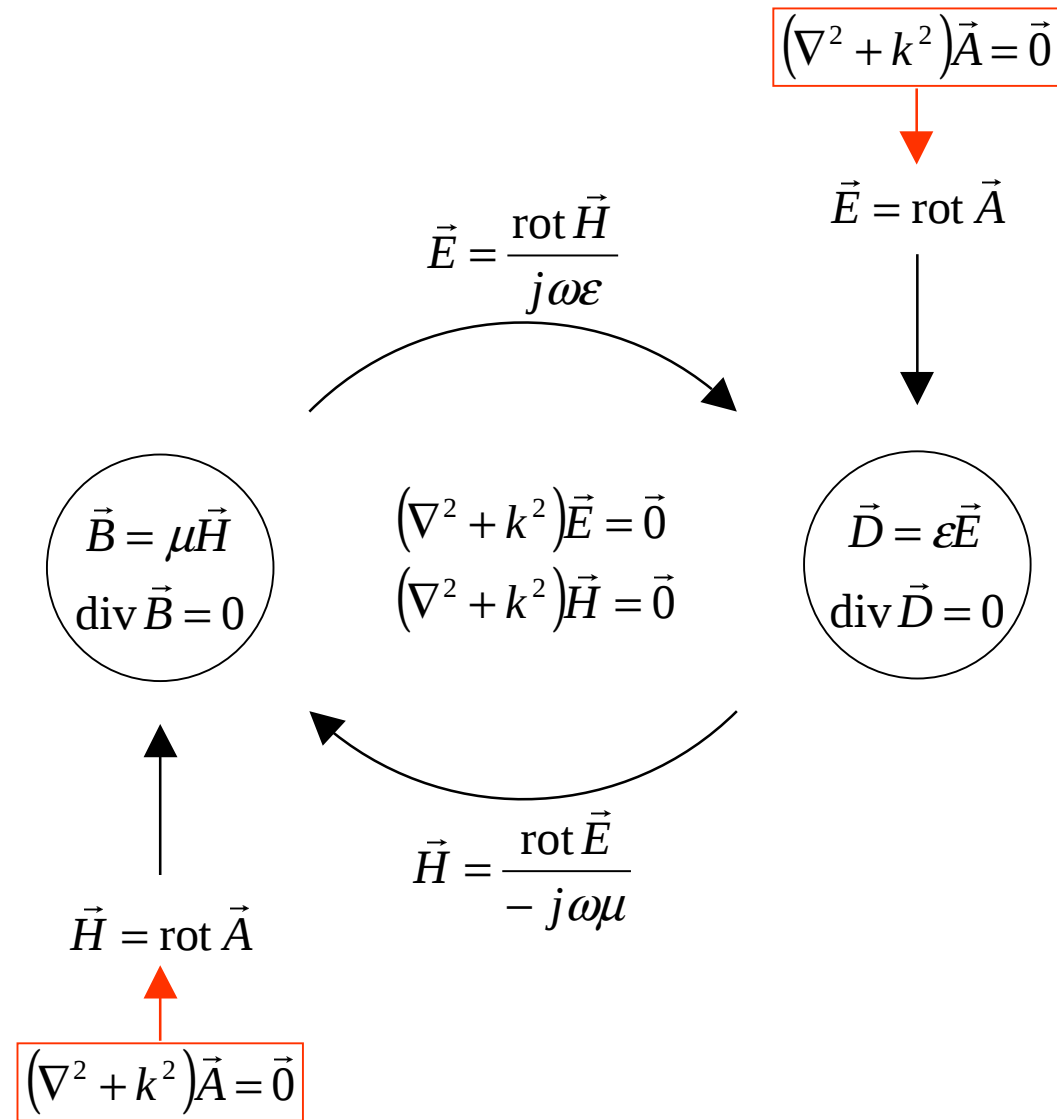
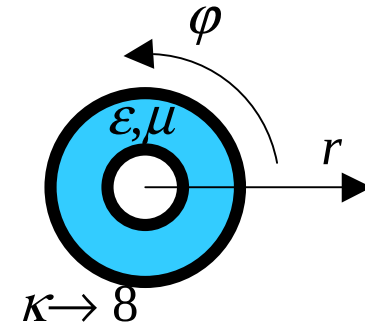
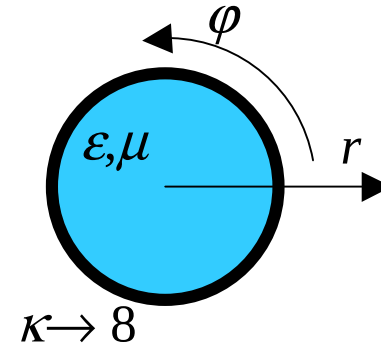
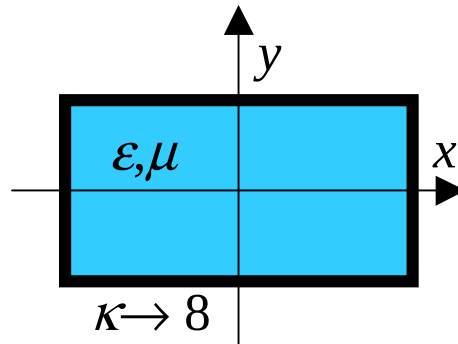


linear, isotrop, homogen, zeitharmonisch, $J = 0$



Wellenleiter (längshomogen)
querhomogen, verlustfrei

z = Längskoordinate
 a, b = Querkoordinaten



TE - Wellen (H - Wellen): $\vec{E} = \text{rot } \vec{A}^H$ mit $\vec{A}^H(a, b, z) = A^H(a, b, z)\vec{e}_z$

TM - Wellen (E - Wellen): $\vec{H} = \text{rot } \vec{A}^E$ mit $\vec{A}^E(a, b, z) = A^E(a, b, z)\vec{e}_z$

$$\vec{A}(a, b, z) = A_t(a, b) \exp(-jk_z z) \vec{e}_z$$

$$(\nabla^2 + k^2)A = 0 \rightarrow (\nabla_t^2 + k^2 - k_z^2)A_t = 0$$

2D Eigenwertproblem

$$\begin{aligned} (\nabla_t^2 + k_c^2)A_t &= 0 \\ &+ \text{RB} \end{aligned}$$

Wellenleitermoden (Eigenvektoren): $A_t(a, b)$

Grenzwellenzahlen (Eigenwerte): k_c

$$(\nabla^2 + k_c^2)A = 0$$

+RB

$$k_c > 0$$

$$\omega_c = ck_c$$

$$k_z = -j\sqrt{k_c^2 - k^2}$$

TE-Wellen
H-Wellen

$$\vec{E} = \text{rot}(A^H(a, b)\vec{e}_z)e^{-jk_z z}$$

$$\vec{H} = \frac{1}{Z_F^H}\vec{e}_z \times \vec{E} + \frac{jk_c^2}{\omega\mu}A^H(a, b)e^{-jk_z z}\vec{e}_z$$

$$Z_F^H = \sqrt{\mu/\varepsilon} \frac{k}{k_z}$$

TM-Wellen
E-Wellen

$$\vec{H} = \text{rot}(A^E(a, b)\vec{e}_z)e^{-jk_z z}$$

$$\vec{E} = Z_F^E\vec{H} \times \vec{e}_z - \frac{jk_c^2}{\omega\varepsilon}A^E(a, b)e^{-jk_z z}\vec{e}_z$$

$$Z_F^E = \sqrt{\mu/\varepsilon} \frac{k_z}{k}$$

$$k_c = 0$$

$$k_z = k$$

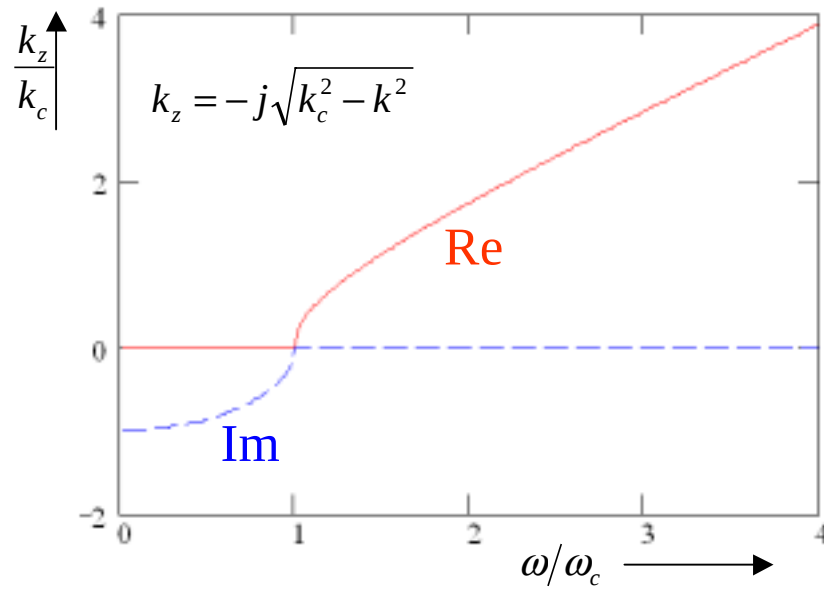
TEM-Wellen

$$\vec{E} = \text{rot}(A^H(a, b)\vec{e}_z)e^{-jk_z z} = Z_F\vec{H} \times \vec{e}_z$$

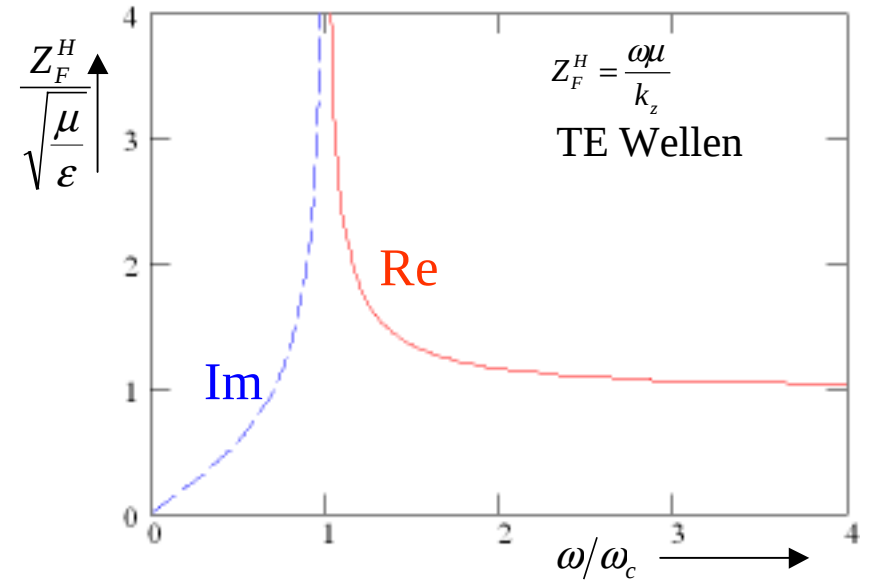
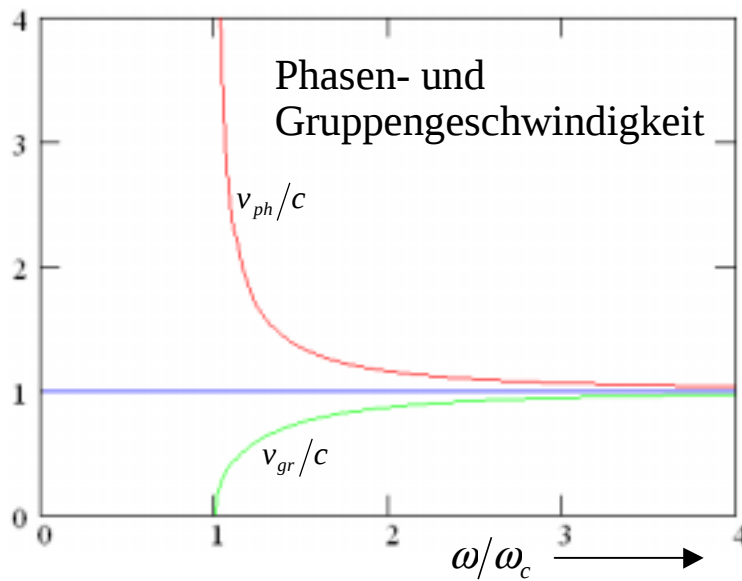
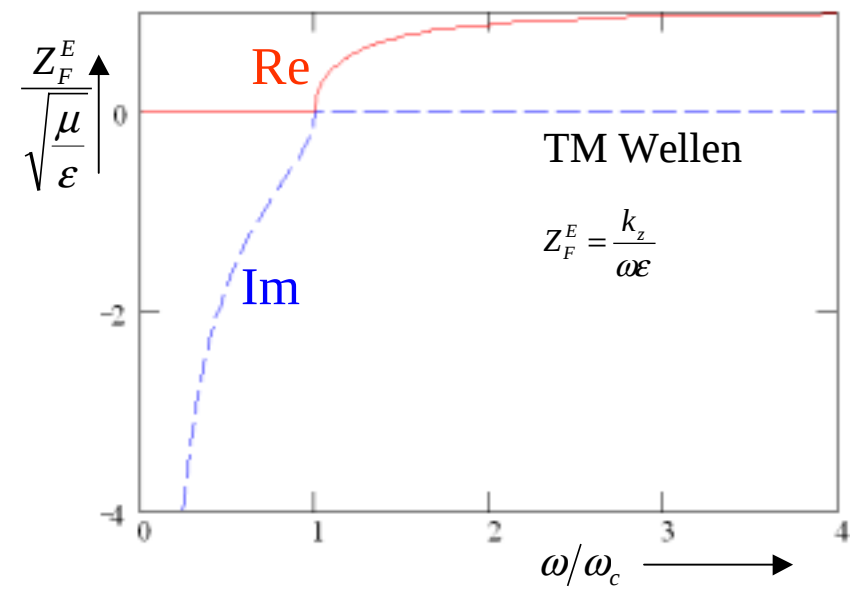
$$\vec{H} = \text{rot}(A^E(a, b)\vec{e}_z)e^{-jk_z z} = \frac{1}{Z_F}\vec{e}_z \times \vec{E}$$

$$Z_F = \sqrt{\mu/\varepsilon}$$

Wellenzahl

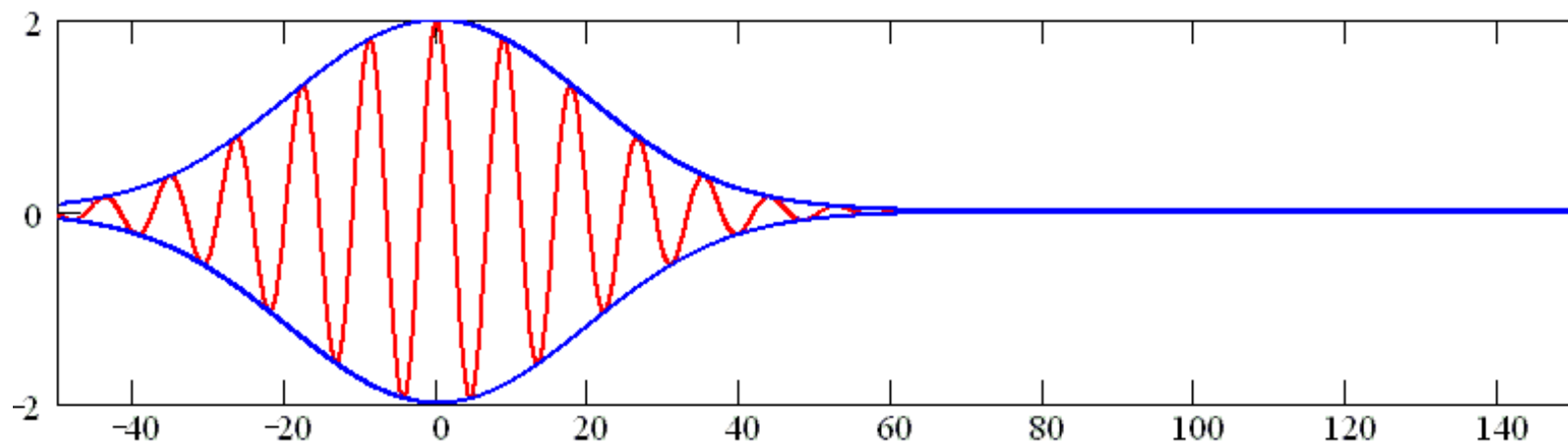
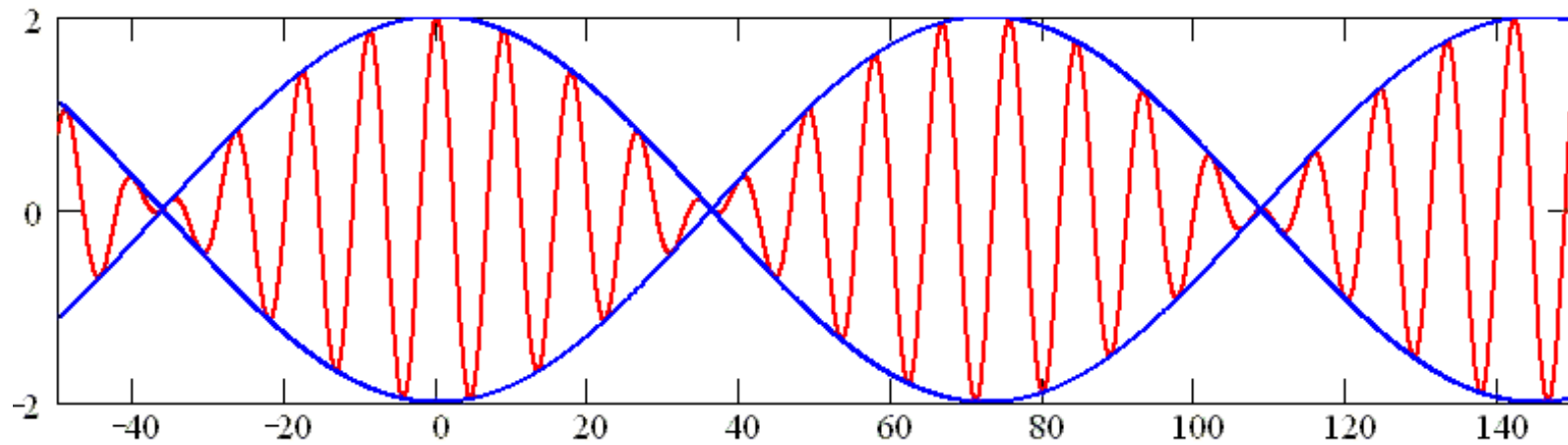


Feldwellenwiderstand

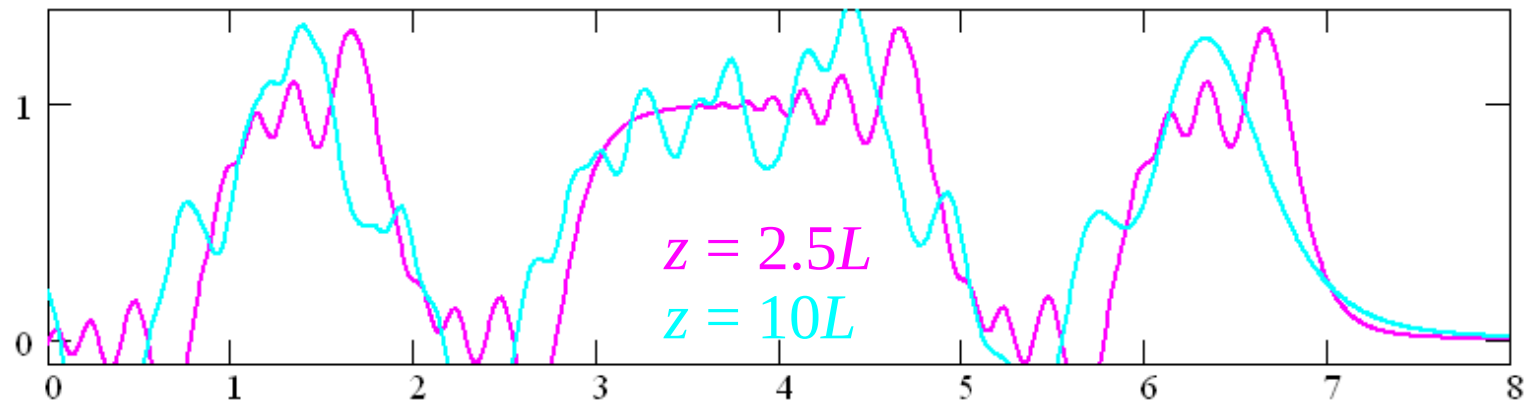
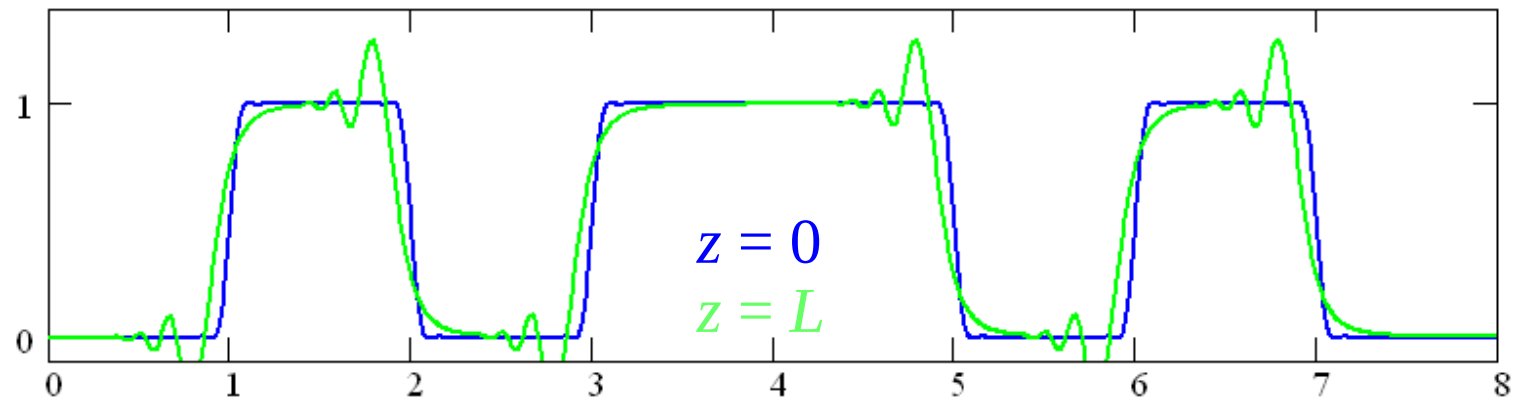


Phasengeschwindigkeit: $v_{\text{ph}} = (k(\omega)/\omega)^{-1}$

Gruppengeschwindigkeit: $v_{\text{ph}} = (dk(\omega)/d\omega)^{-1}$



Dispersion: Hüllkurve



Energiegeschwindigkeit
TE- oder H-Moden

$$\vec{E} = \begin{pmatrix} \partial A_t / \partial y \\ -\partial A_t / \partial x \\ 0 \end{pmatrix} \cos(\omega t - k_z z)$$

$$\vec{H} = \frac{1}{Z_F^H} \vec{e}_z \times \vec{E} - \frac{k_c^2}{\omega \mu} A_t(a, b) \sin(\omega t - k_z z) \vec{e}_z$$

$$S_z = (\vec{E} \times \vec{H}) \cdot \vec{e}_z$$

$$S_z = \frac{1}{Z_F^H} \left((\partial A_t / \partial x)^2 + (\partial A_t / \partial y)^2 \right) \cos^2(\omega t - k_z z)$$

$$\overline{S_z} = \frac{1}{2Z_F^H} \left((\partial A_t / \partial x)^2 + (\partial A_t / \partial y)^2 \right)$$

$$w = \frac{1}{2} (\epsilon \vec{E}^2 + \mu \vec{H}^2)$$

$$w = \frac{\epsilon}{2} \left(1 + \left(\frac{Z}{Z_F^H} \right)^2 \right) \left((\partial A_t / \partial x)^2 + (\partial A_t / \partial y)^2 \right) \cos^2(\omega t - k_z z) + \frac{\epsilon}{2} \left(\frac{k_c^2}{k} A_t \right)^2 \sin^2(\omega t - k_z z)$$

$$\overline{w} = \frac{\epsilon}{4} \left(1 + \left(\frac{k_z}{k} \right)^2 \right) \left((\partial A_t / \partial x)^2 + (\partial A_t / \partial y)^2 \right) + \frac{\epsilon}{4} \left(\frac{k_c^2}{k} A_t \right)^2$$

Greensche Formel

$$\int_V (\Delta U_1) U_2 dV = \int_{\partial V} U_2 \text{grad} U_1 d\vec{A} - \int_V \text{grad} U_1 \text{grad} U_2 dV$$

$$U_1 = U_2 = A_t ; \quad (\Delta + k_c^2) A_t = 0$$

$$-k_c^2 \int_V A_t^2 dV = \int_{\partial V} A_t \text{grad} A_t d\vec{A} - \int_V \text{grad} A_t \text{grad} A_t dV$$

2D+RB

$$U_1 = U_2 = A_t ; \quad (\Delta + k_c^2) A_t = 0$$

$$k_c^2 \int_A A_t^2 dA = \int_A ((\partial A_t / \partial x)^2 + (\partial A_t / \partial y)^2) dA$$

$$\int_A \overline{w} dA = \frac{\varepsilon}{4} \left(1 + \left(\frac{k_z}{k} \right)^2 \right) \int_A ((\partial A_t / \partial x)^2 + (\partial A_t / \partial y)^2) dA + \frac{\varepsilon}{4} \left(\frac{k_c^2}{k} \right)^2 \int_A A_t^2 dA$$

$$\int_A \overline{w} dA = \frac{\varepsilon}{4} \left(1 + \frac{k_z^2}{k^2} + \frac{k_c^2}{k^2} \right) \int_A ((\partial A_t / \partial x)^2 + (\partial A_t / \partial y)^2) dA$$

$$\int_A \overline{w} dA = \frac{\varepsilon}{2} \int_A \dots dA$$

$$\int_A \overline{S_z} dA = \frac{1}{2Z_F^H} \int_A \dots dA$$

$$v_{\text{energy}} = \frac{P}{W'} = \frac{\int_A \overline{S_z} dA}{\int_A \overline{w} dA} = c \frac{k_z}{k}$$