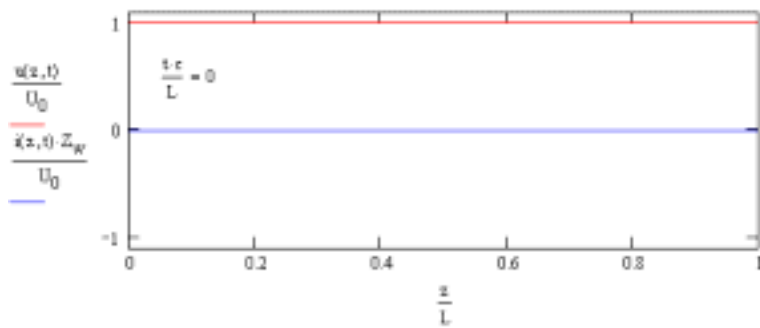
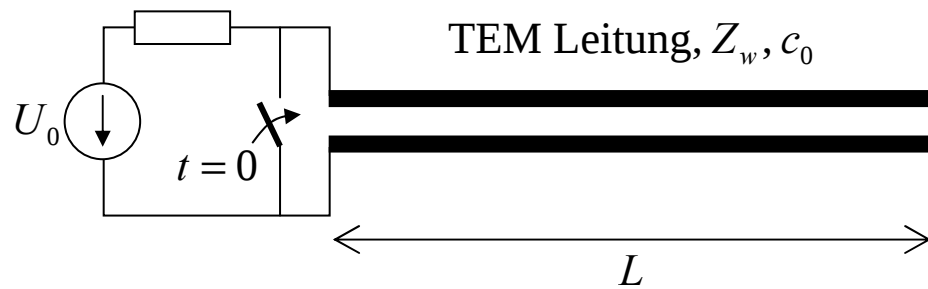


$$\nabla_t^2 A_t = -k_c^2 A_t$$

kartesische Lösungen $A_t(x, y)$

$$A_t(x, y) = f(x)g(y)$$

$$\boxed{\frac{d^2 f(x)}{dx^2}} + \boxed{\frac{\partial^2 g(y)}{\partial x^2}} = -k_c^2$$

$$\quad \quad \quad = -k_x^2 \quad \quad = -k_y^2$$

$$f''(x) + k_x^2 f(x) = 0$$

$$g''(y) + k_y^2 g(y) = 0$$

$$A_t(x, y) = C \begin{Bmatrix} \cos k_x x \\ \sin k_x x \end{Bmatrix} \begin{Bmatrix} \cos k_y y \\ \sin k_y y \end{Bmatrix}$$

kreiszyklindrische Lösungen $A_t(r, \varphi)$

$$A_t(r, \varphi) = f(r)g(\varphi)$$

$$\boxed{\frac{r^2 f'' + rf' + r^2 k_c^2 f}{f}} + \boxed{\frac{g''}{g}} = 0$$

$$\quad \quad \quad = m^2 \quad \quad = -m^2$$

$$f(r) = Z(x = k_c r) \quad x^2 Z'' + xZ' + (x^2 + m^2)Z = 0$$

Besselsche DGL

$$g''(\varphi) + m^2 g(\varphi) = 0$$

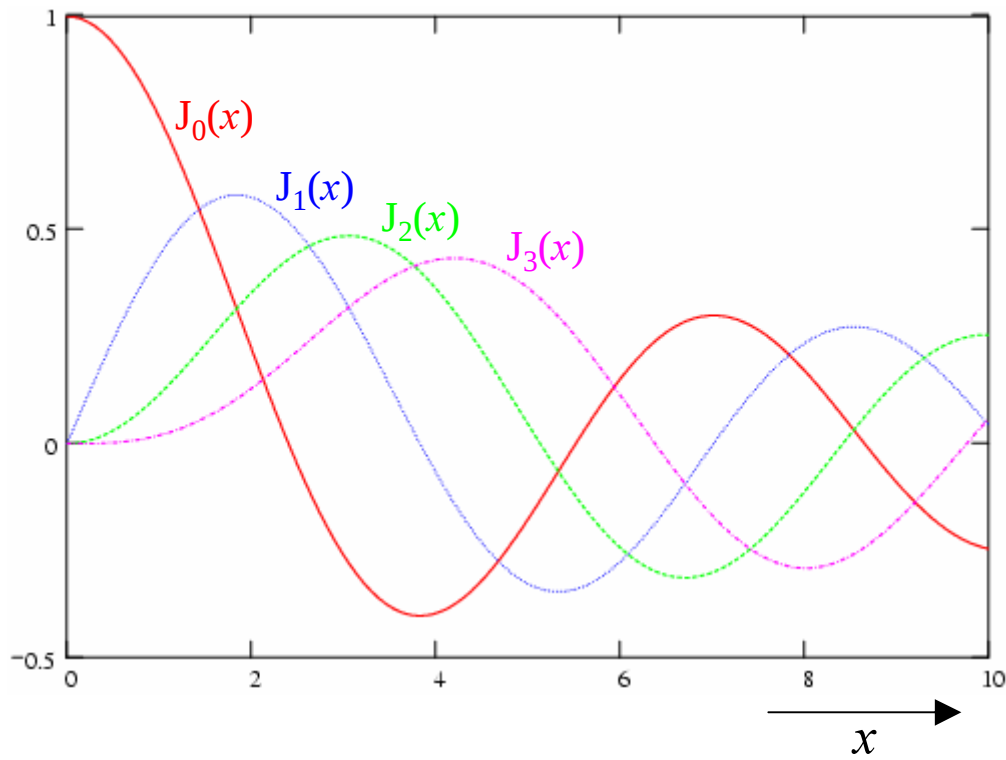
$$A_t(r, \varphi) = C \begin{Bmatrix} Z_m^{(1)}(k_c r) \\ Z_m^{(2)}(k_c r) \end{Bmatrix} \begin{Bmatrix} \cos m\varphi \\ \sin m\varphi \end{Bmatrix}$$

Randbedingungen (metallische Wände)

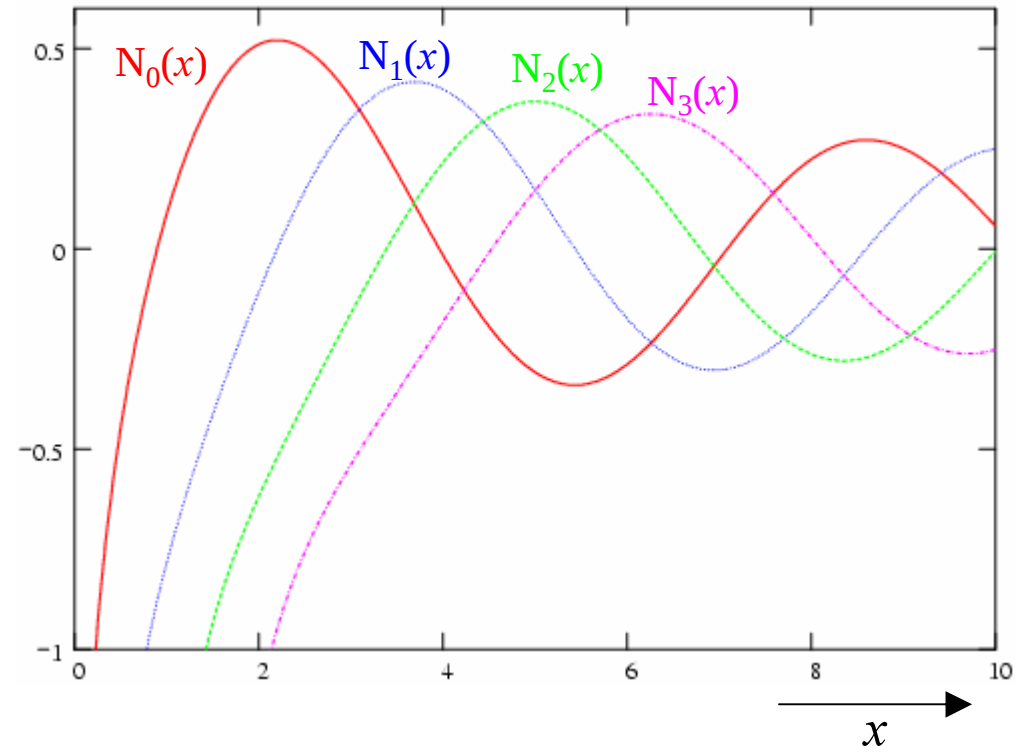
	TE-Wellen	TM-Wellen
Rand $\overset{r}{x} = const$	$f'(\overset{r}{x}) = 0$	$f(\overset{r}{x}) = 0$
Rand $\overset{\varphi}{y} = const$	$g'(\overset{\varphi}{y}) = 0$	$g(\overset{\varphi}{y}) = 0$

Zylinder Funktionen

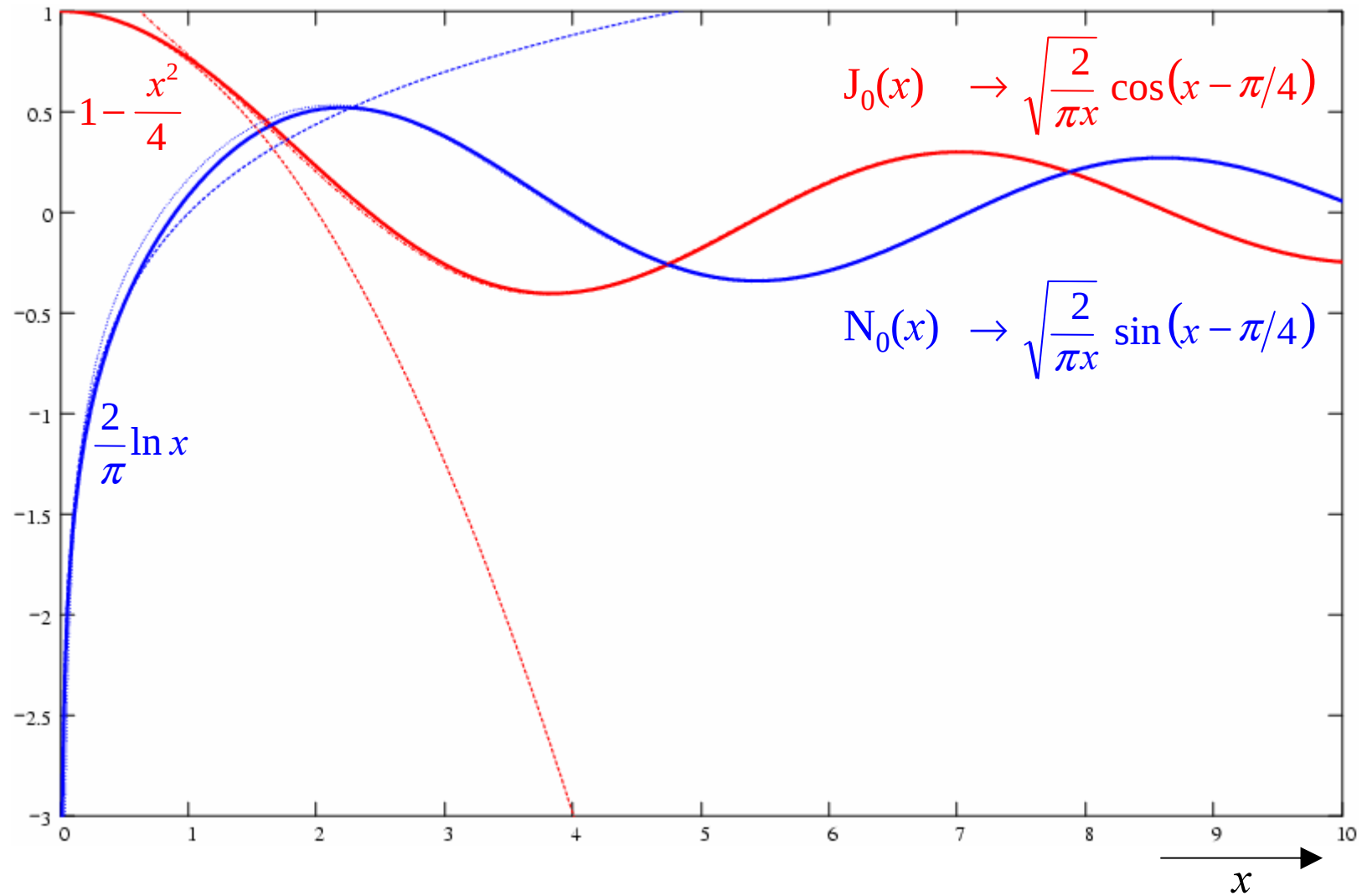
Besselfunktionen



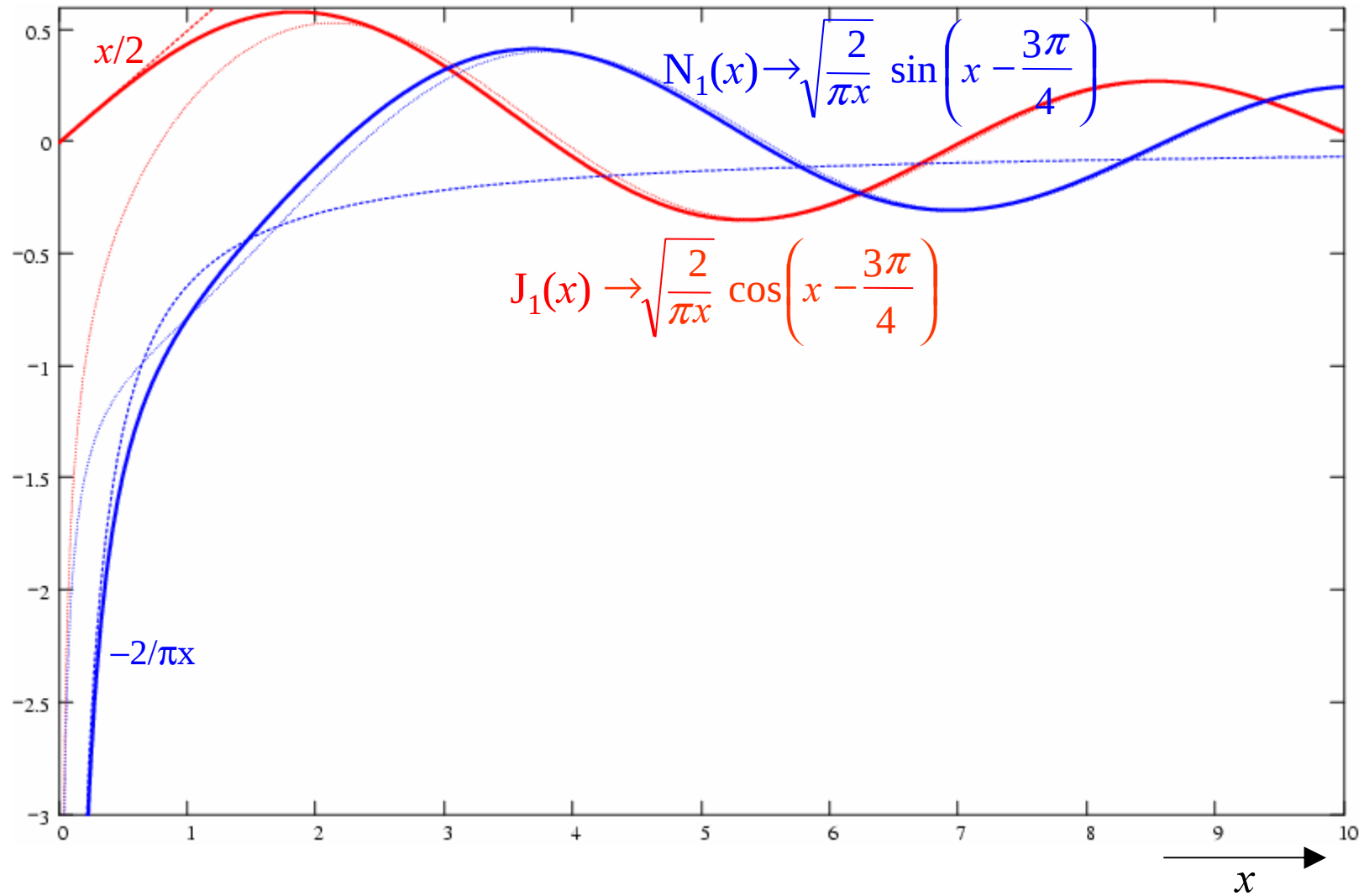
Neumannfunktionen



azimuthale Ordnung $m=0$ (Monopole Moden)



azimuthale Ordnung $m=1$ (Dipole Moden)



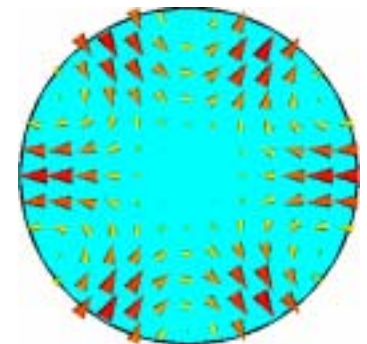
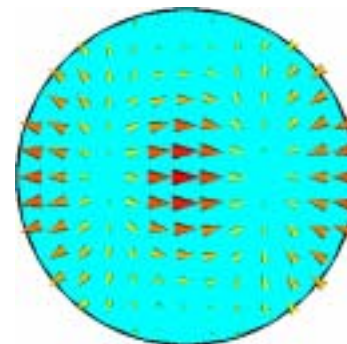
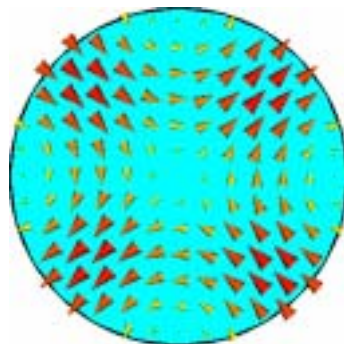
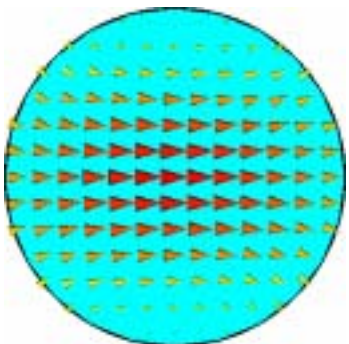
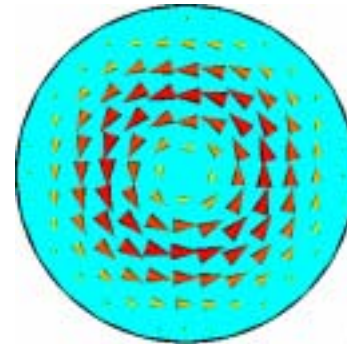
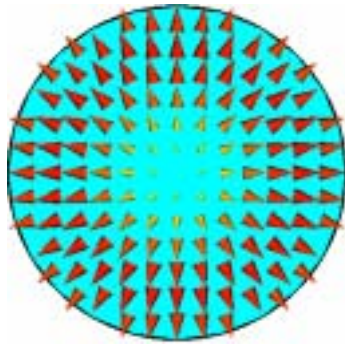
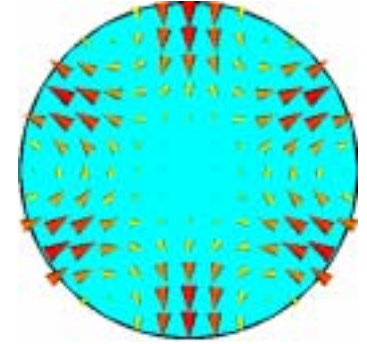
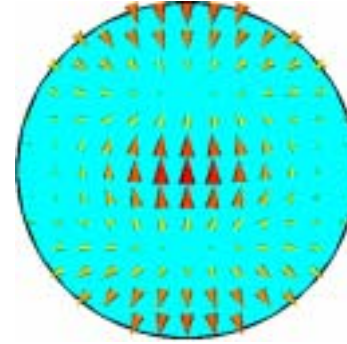
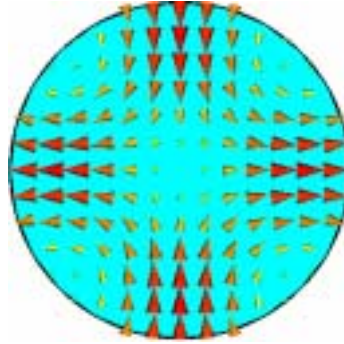
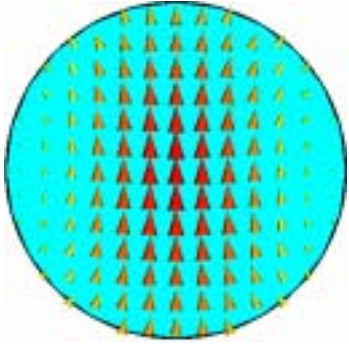
$$f_{c,H11} = \frac{1.841}{2\pi} \frac{c}{R}$$

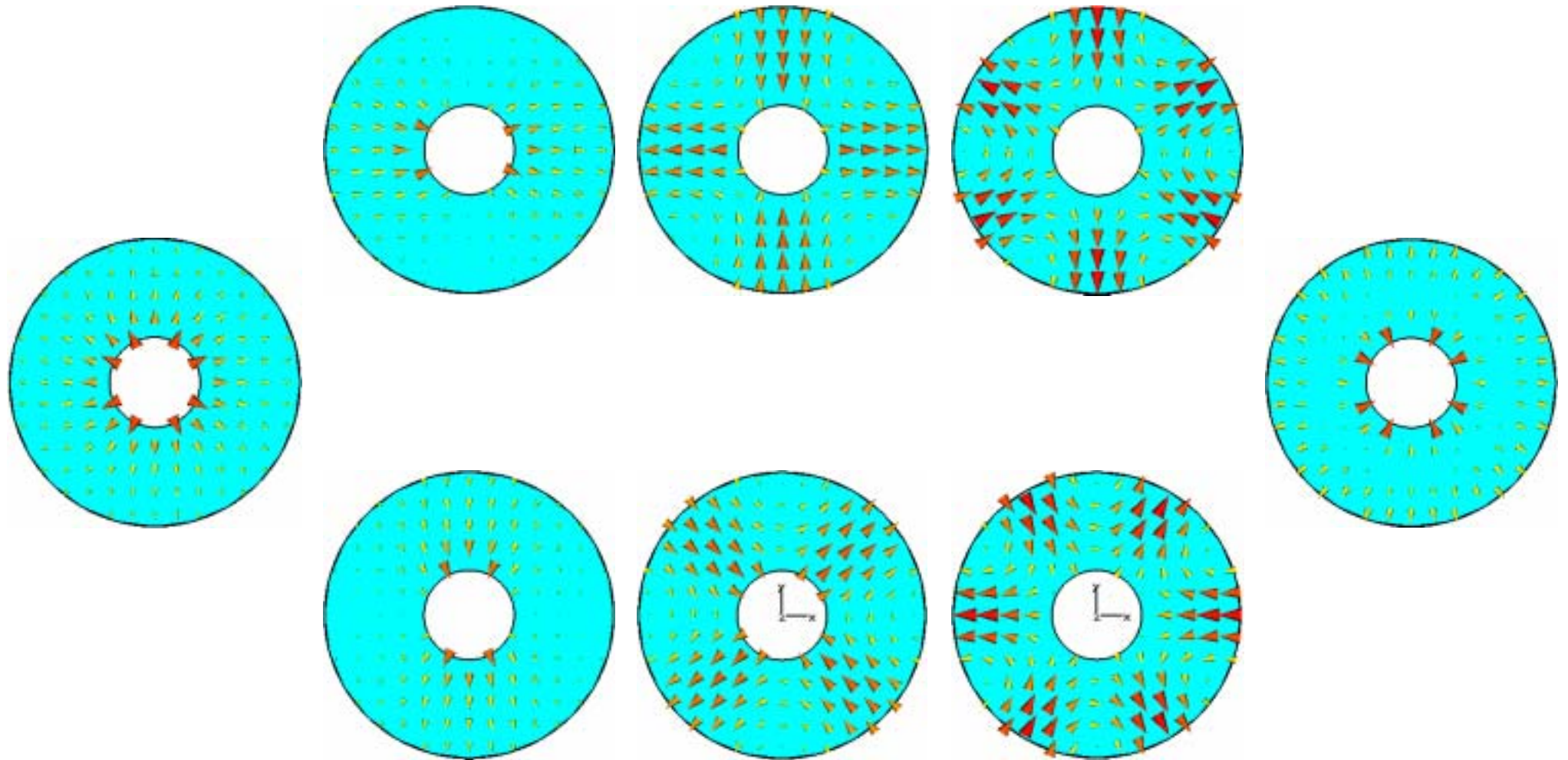
$$f_c = 1.306 f_{c,H11}$$

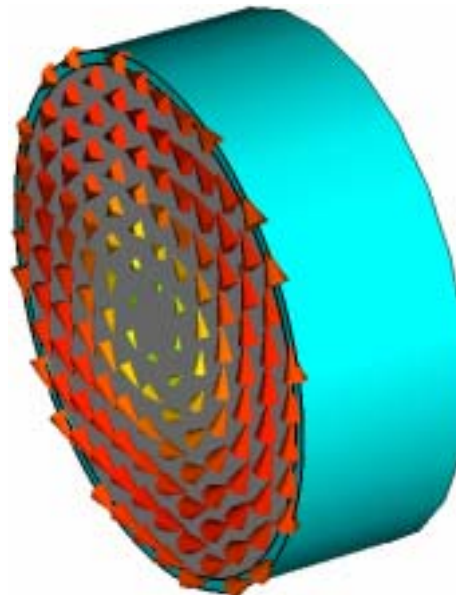
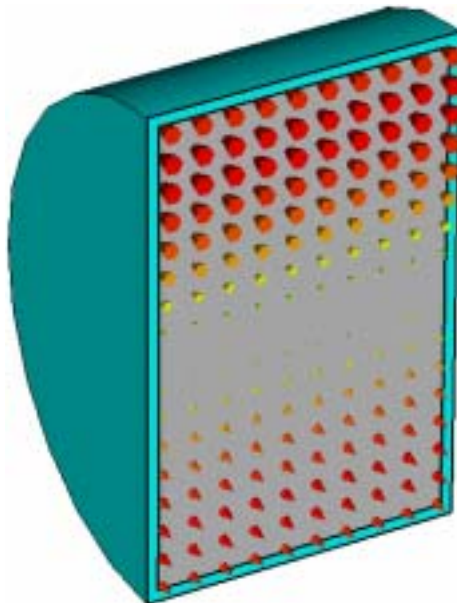
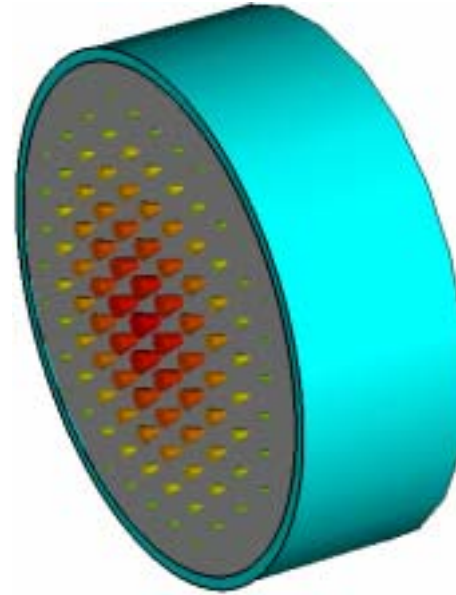
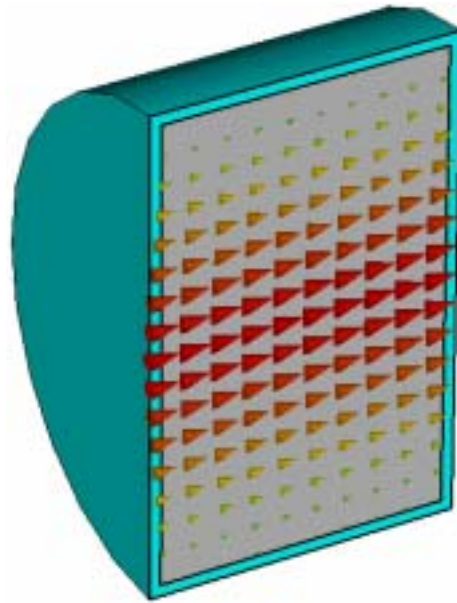
$$f_c = 1.659 f_{c,H11}$$

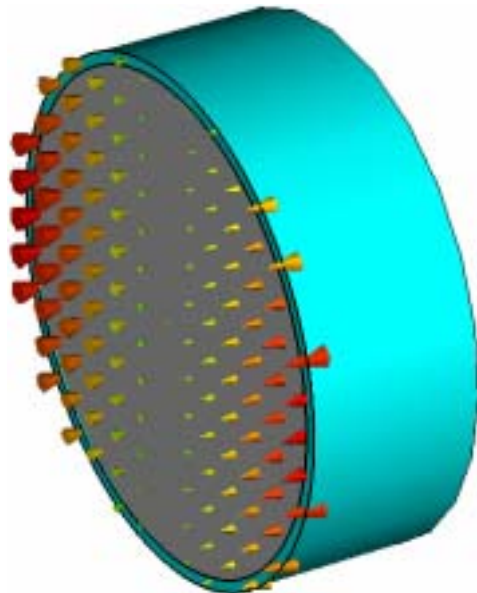
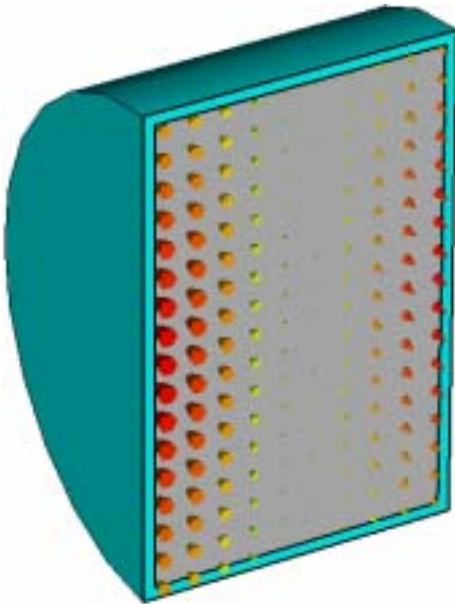
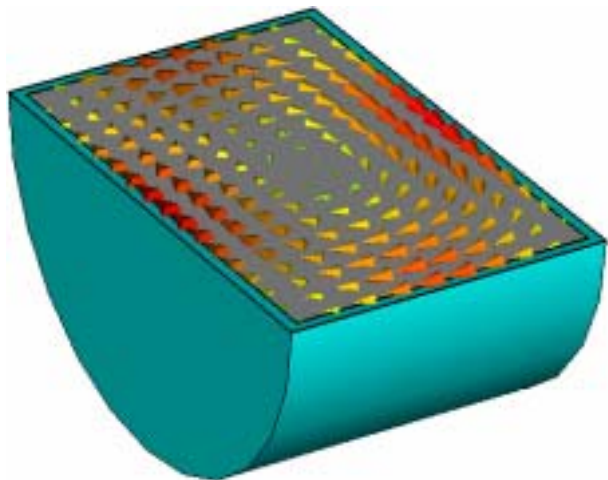
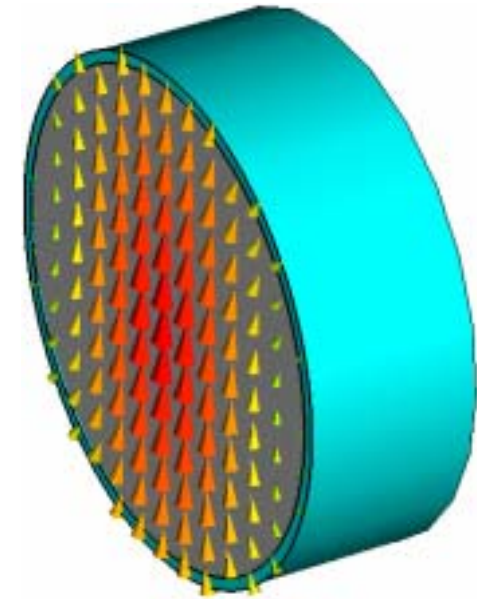
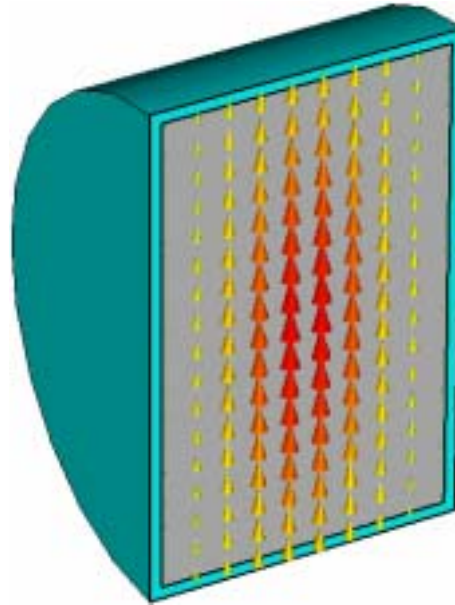
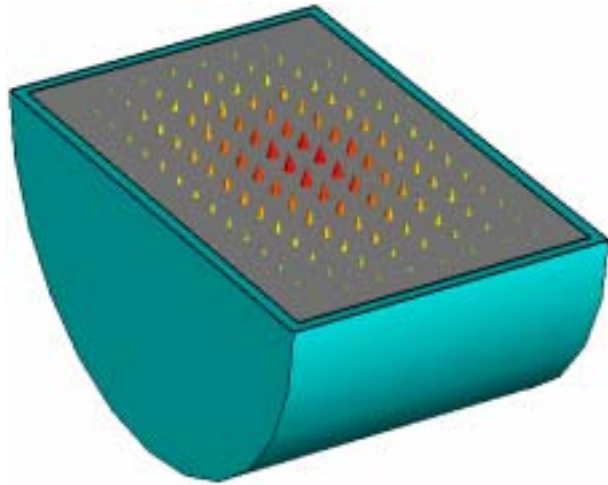
$$f_c = 2.018 f_{c,H11}$$

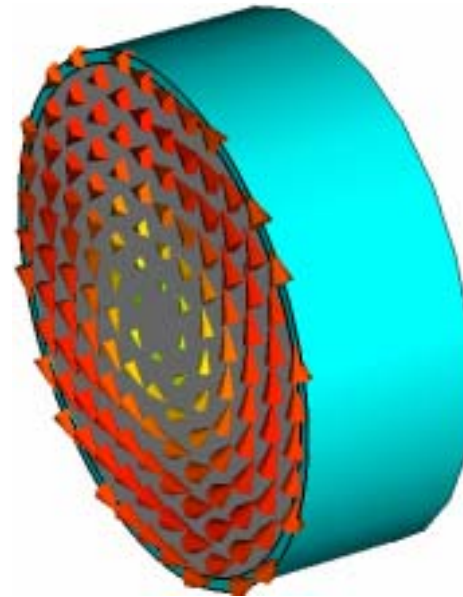
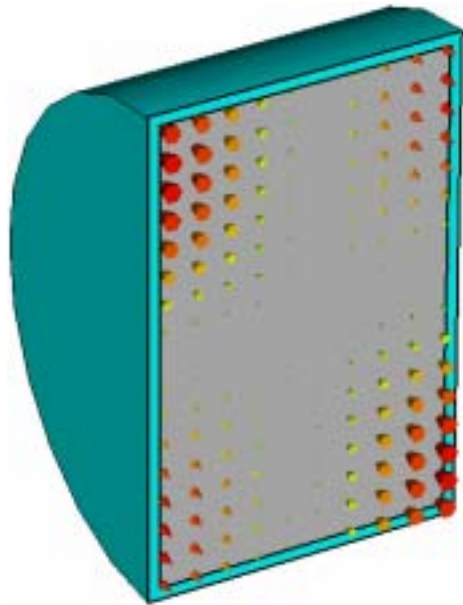
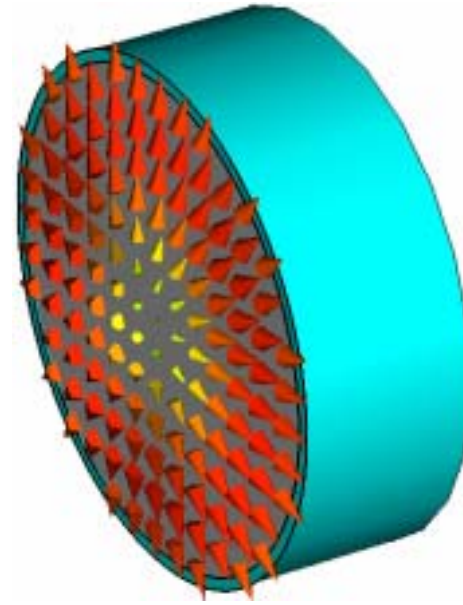
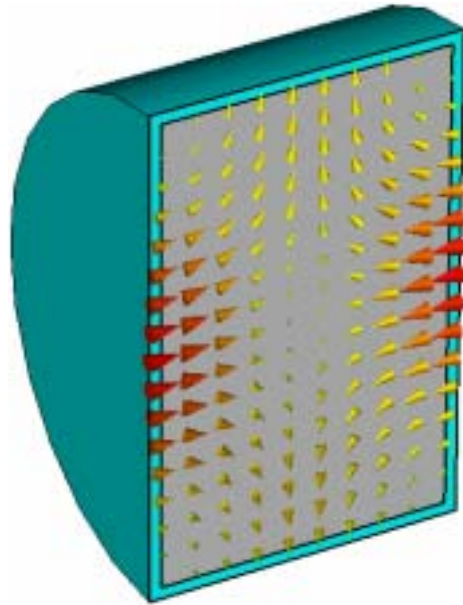
$$f_c = 2.282 f_{c,H11}$$



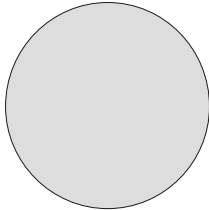
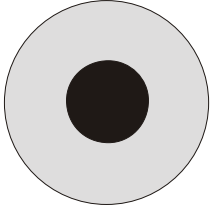
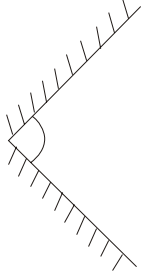






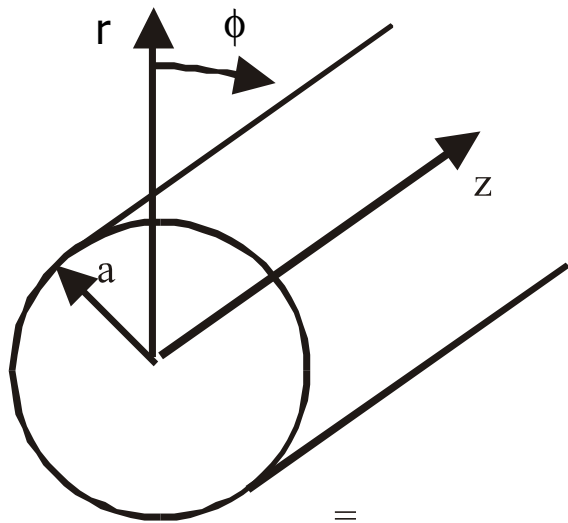


Kriterien zur Auswahl der Zylinderfunktionen

	Rundhohlleiter	Koaxialleiter	Sektorhorn
			
M	M ganzzahlig		M ist vom Winkel 'alpha' Abhängig
Zylinder-Funktion	J_M	$AJ_M^+ BN_M$	Einstrahlung: H_M Ausstrahlung: H_M

Wellentypen im Rundhohlleiter

Welle in +z-Richtung:



$$\vec{A} = A \vec{e}_z \quad \text{mit}$$

$$A = C J_m(Kr) \cos(m\varphi) e^{-jk_z z}$$

und

$$k_z = \sqrt{k^2 - K^2}$$

Nullstellen der Besselfunktion

		m				
n	jmn	0	1	2	3	
	1	2.405	3.832	5.136	6.380	Nullstellen
	2	5.520	7.016	8.417	9.761	von
	3	8.654	10.173	11.620	13.015	J_{mn}

Nullstellen der 1. Ableitung der Besselfunktion

		m				
n	j' _{mn}	0	1	2	3	
	1	3.832	1.841	3.054	4.201	Nullstellen
	2	7.016	5.331	6.706	8.015	von
	3	10.173	8.536	9.969	11.346	J' _{mn} (Ableitung v. J _{mn})

TM_{zmn}-Wellen

$$H_r = -C \frac{m}{r} J_m \left(j_{mn} \frac{r}{a} \right) \sin m \varphi e^{-jk_z z}$$

$$H_\varphi = -C \frac{j_{mn}}{a} J'_m \left(j_{mn} \frac{r}{a} \right) \cos m \varphi e^{-jk_z z}$$

$$E_r = -C Z^E \frac{j_{mn}}{a} J'_m \left(j_{mn} \frac{r}{a} \right) \cos m \varphi e^{-jk_z z}$$

$$E_\varphi = C Z^E \frac{m}{r} J_m \left(j_{mn} \frac{r}{a} \right) \sin m \varphi e^{-jk_z z}$$

$$E_z = C \left(\frac{j_{mn}}{a} \right)^2 \frac{1}{j\omega\epsilon} J_m \left(j_{mn} \frac{r}{a} \right) \cos m \varphi e^{-jk_z z}$$

$$\text{mit der Separationsgleichung } k_z^2 = k^2 - \left(\frac{j_{mn}}{a} \right)^2$$

TE_{zmn}-Wellen

$$E_r = -C \frac{m}{r} J_m \left(j'_{mn} \frac{r}{a} \right) \sin m\varphi e^{-jk_z z}$$

$$E_\varphi = -C \frac{j'_{mn}}{a} J'_m \left(j'_{mn} \frac{r}{a} \right) \cos m\varphi e^{-jk_z z}$$

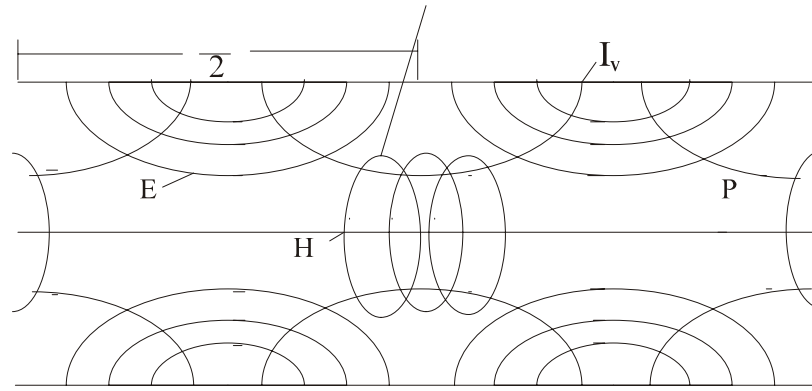
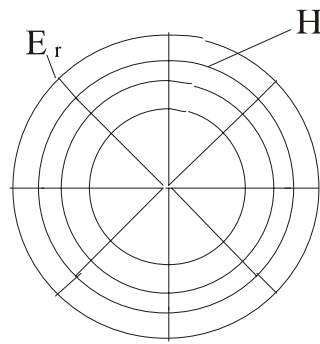
$$H_r = C \frac{1}{Z^H} \frac{j'_{mn}}{a} J'_m \left(j'_{mn} \frac{r}{a} \right) \cos m\varphi e^{-jk_z z}$$

$$H_\varphi = -C \frac{1}{Z^H} \frac{m}{r} J_m \left(j'_{mn} \frac{r}{a} \right) \sin m\varphi e^{-jk_z z}$$

$$H_z = -C \left(\frac{j'_{mn}}{a} \right)^2 \frac{1}{j\omega\mu} J_m \left(j'_{mn} \frac{r}{a} \right) \cos m\varphi e^{-jk_z z}$$

mit der Separationsgleichung $k_z^2 = k^2 - \left(\frac{j'_{mn}}{a} \right)^2$

TM_{z01}-Welle



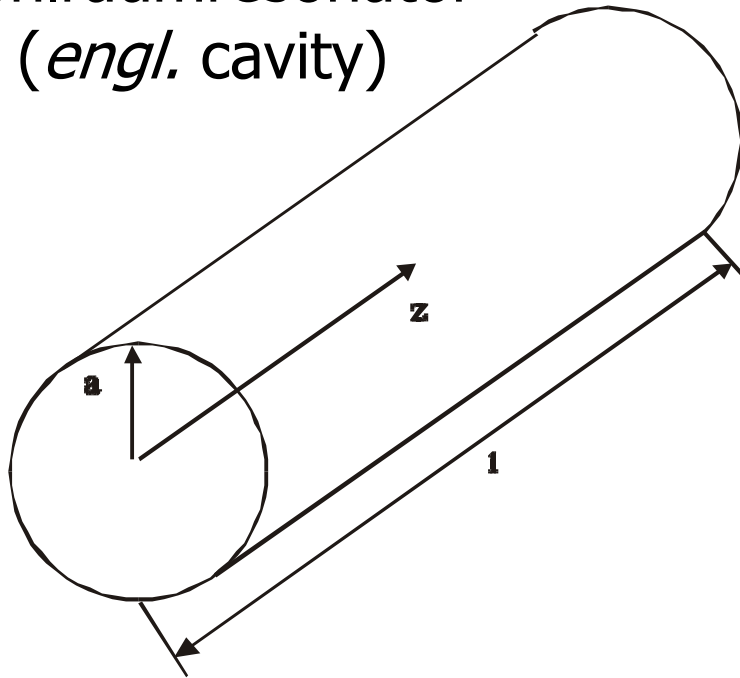
$$H_{\varphi} = -C \frac{j_{01}}{a} J'_{01} \left(j_{01} \frac{r}{a} \right) e^{-jk_z z}$$

$$E_r = -C Z^E \frac{j_{01}}{a} J'_{01} \left(j_{01} \frac{r}{a} \right) e^{-jk_z z}$$

$$E_z = C \left(\frac{j_{01}}{a} \right)^2 \frac{1}{j\omega\epsilon} J_0 \left(j_{01} \frac{r}{a} \right) e^{-jk_z z}$$

Kreiszyklindrische Resonatoren

Hohlraumresonator
(*engl. cavity*)



Stehwelle auch in z-Richtung

$$k_z = \frac{p\pi}{L}$$

Resonanzfrequenzen der
Eigenschwingungen (Moden):

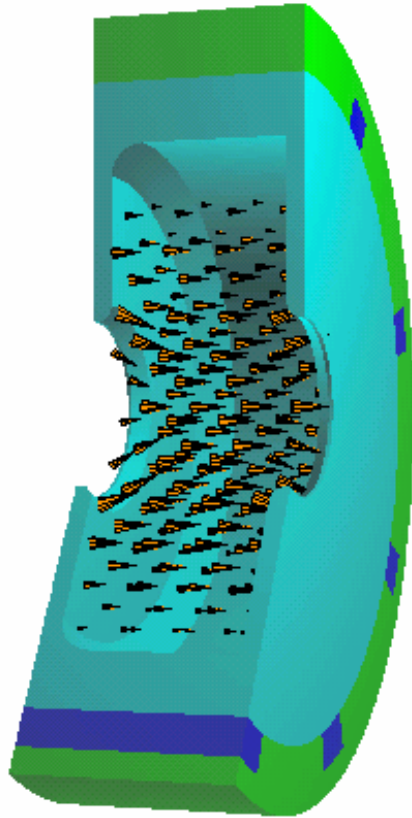
TM_z-Wellen:

$$\omega_{mnp} = c \sqrt{\left(\frac{j_{mn}}{a}\right)^2 + \left(\frac{p\pi}{L}\right)^2}$$

TE_z-Wellen:

$$\omega_{mnp} = c \sqrt{\left(\frac{j'_{mn}}{a}\right)^2 + \left(\frac{p\pi}{L}\right)^2}$$

TM₀₁₀-Mode und die Beschleunigung von Elektronen



SBLC: $f = 3$ GHz

TESLA: $f = 1.3$ GHz

