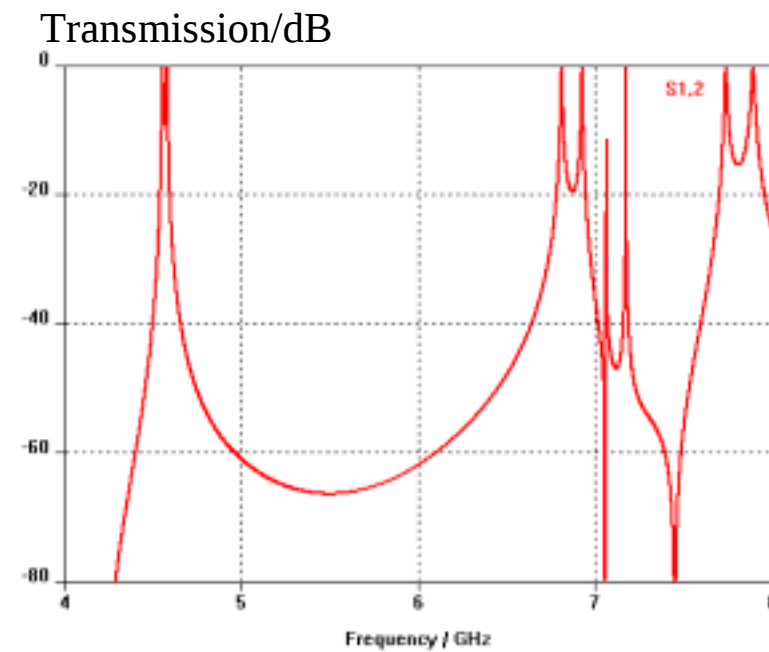
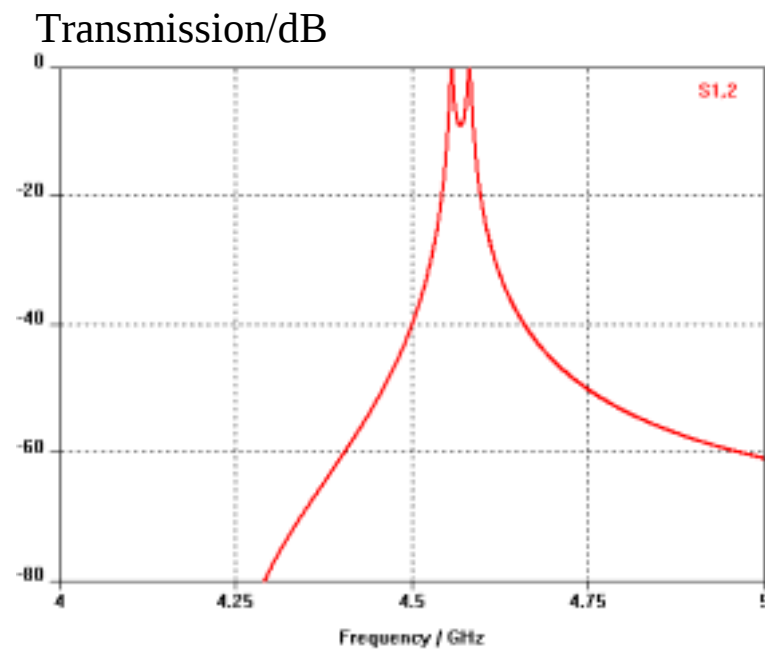
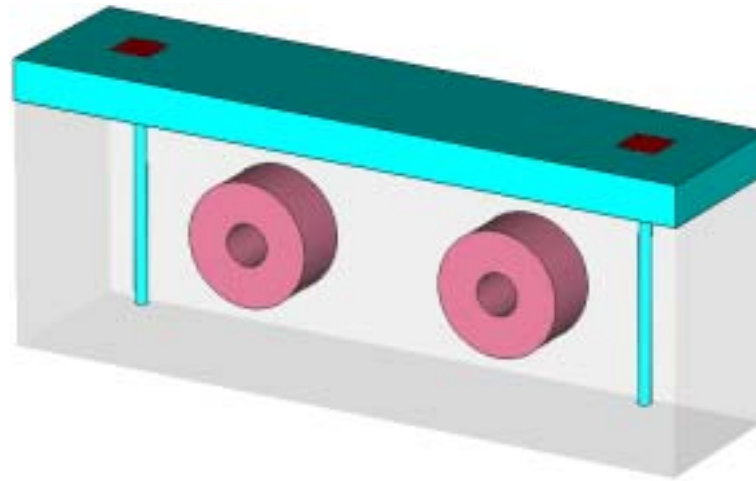
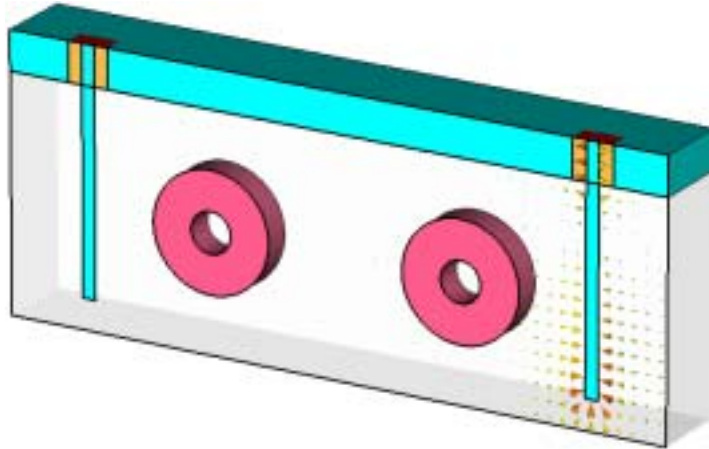
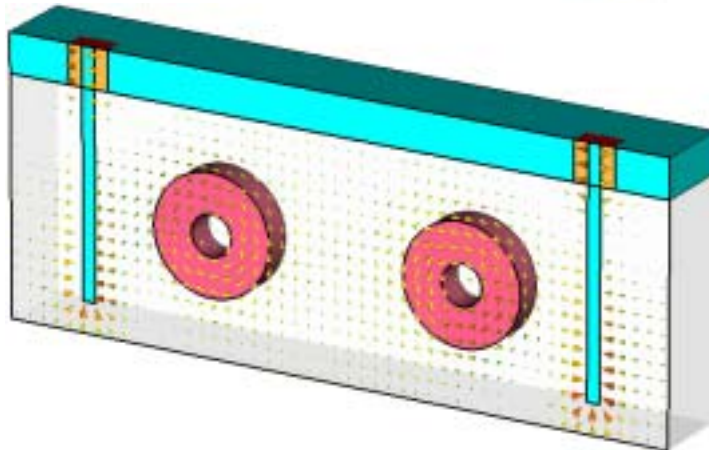




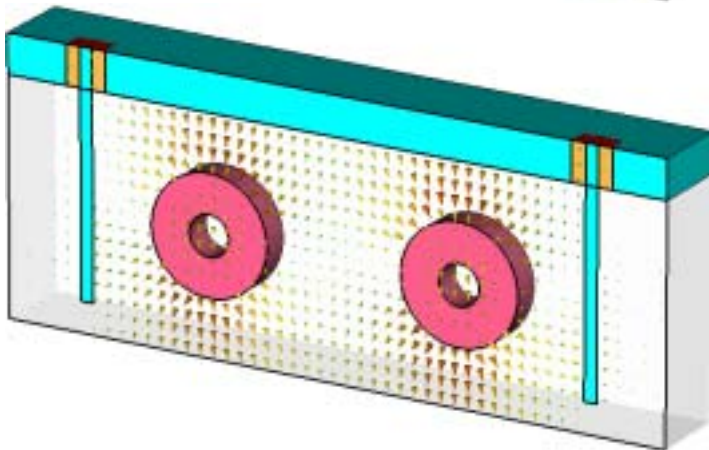
v = 4



v = 5



v = 7



einige Moden
(Koaxial-Tore: PMC Rand)

komplexe Schreibweise:

$$-j\omega_v\mu \underline{\vec{H}}_v = \text{rot } \underline{\vec{E}}_v$$

$$j\omega_v\epsilon \underline{\vec{E}}_v = \text{rot } \underline{\vec{H}}_v$$

$$\vec{E}(\vec{r}, t) = \text{Re}\{\underline{\vec{E}}_v \exp(j\omega_v t)\}$$

$$\vec{H}(\vec{r}, t) = \text{Re}\{\underline{\vec{H}}_v \exp(j\omega_v t)\}$$

reelle Schreibweise (oBdA):

$$\vec{E}_v = \text{Re}\{\underline{\vec{E}}_v\}$$

$$\vec{H}_v = \text{Re}\{j\underline{\vec{H}}_v\}$$

$$\vec{E}(\vec{r}, t) = \vec{E}_v \cos(\omega_v t)$$

$$\vec{H}(\vec{r}, t) = \vec{H}_v \sin(\omega_v t)$$

Energie im verlustfreien Resonator

$$\vec{E}(\vec{r}, t) = \vec{E}_v \cos(\omega_v t)$$

$$\vec{H}(\vec{r}, t) = \vec{H}_v \sin(\omega_v t)$$

$$\begin{aligned} W_{tot} &= \frac{1}{2} \int_V \epsilon \|\vec{E}_v\|^2 \cos^2(\omega_v t) dV + \frac{1}{2} \int_V \mu \|\vec{H}_v\|^2 \sin^2(\omega_v t) dV \\ &= \frac{1}{4} \int_V \epsilon \|\vec{E}_v\|^2 dV + \frac{1}{4} \int_V \mu \|\vec{H}_v\|^2 dV + \frac{1}{4} \left(\int_V \epsilon \|\vec{E}_v\|^2 dV - \int_V \mu \|\vec{H}_v\|^2 dV \right) \cos(2\omega_v t) dV \end{aligned}$$

$$\cos^2(\omega_v t) = \frac{1}{2} (1 + \cos(2\omega_v t))$$

$$\sin^2(\omega_v t) = \frac{1}{2} (1 + \cos(2\omega_v t))$$

$$W_{tot} = \underbrace{\frac{1}{4} \int_V \epsilon \|\vec{E}_v\|^2 dV}_{\bar{W}_E} + \underbrace{\frac{1}{4} \int_V \mu \|\vec{H}_v\|^2 dV}_{\bar{W}_M}$$

Orthogonalität von Moden (Motivation)

$$\vec{E}(\vec{r}, t) = \vec{E}_1 \cos(\omega_1 t) + \vec{E}_2 \cos(\omega_2 t) \quad \text{Überlagerung von 2 Moden}$$

$$\vec{H}(\vec{r}, t) = \vec{H}_1 \sin(\omega_1 t) + \vec{H}_2 \sin(\omega_2 t)$$

$$W_{tot} = \frac{1}{2} \int_V \epsilon \left\| \vec{E}_1 \cos(\omega_1 t) + \vec{E}_2 \cos(\omega_2 t) \right\|^2 dV + \frac{1}{2} \int_V \mu \left\| \vec{H}_1 \sin(\omega_1 t) + \vec{H}_2 \sin(\omega_2 t) \right\|^2 dV$$

$$W_{tot} = \left[\frac{1}{2} \int_V \epsilon \left\| \vec{E}_1 \right\|^2 \cos^2(\omega_1 t) dV + \frac{1}{2} \int_V \mu \left\| \vec{H}_1 \right\|^2 \sin^2(\omega_1 t) dV \right] +$$

$$\left[\frac{1}{2} \int_V \epsilon \left\| \vec{E}_2 \right\|^2 \cos^2(\omega_2 t) dV + \frac{1}{2} \int_V \mu \left\| \vec{H}_2 \right\|^2 \sin^2(\omega_2 t) dV \right] +$$

$$\frac{2}{2} \int_V \epsilon \vec{E}_1 \cdot \vec{E}_2 \cos(\omega_1 t) \cos(\omega_2 t) dV + \frac{2}{2} \int_V \mu \vec{H}_1 \cdot \vec{H}_2 \sin(\omega_1 t) \sin(\omega_2 t) dV$$

$$W_{tot,1} = W_{tot,1} + W_{tot,2}$$

$$\begin{aligned} \int_V \epsilon \vec{E}_1 \cdot \vec{E}_2 dV &= 0 \\ \int_V \mu \vec{H}_1 \cdot \vec{H}_2 dV &= 0 \end{aligned}$$

falls $\omega_1 \neq \omega_2$
oder Moden orthogonal gewählt

Reziprozität

$$\oint_{\partial V} (\vec{E}_1 \times \vec{H}_2 - \vec{E}_2 \times \vec{H}_1) d\vec{A} = \int_V (\vec{J}_1 \vec{E}_2 - \vec{J}_2 \vec{E}_1) dV$$

geschlossenes System: $\int_V \vec{J}_1 \vec{E}_2 dV = \int_V \vec{J}_2 \vec{E}_1 dV$

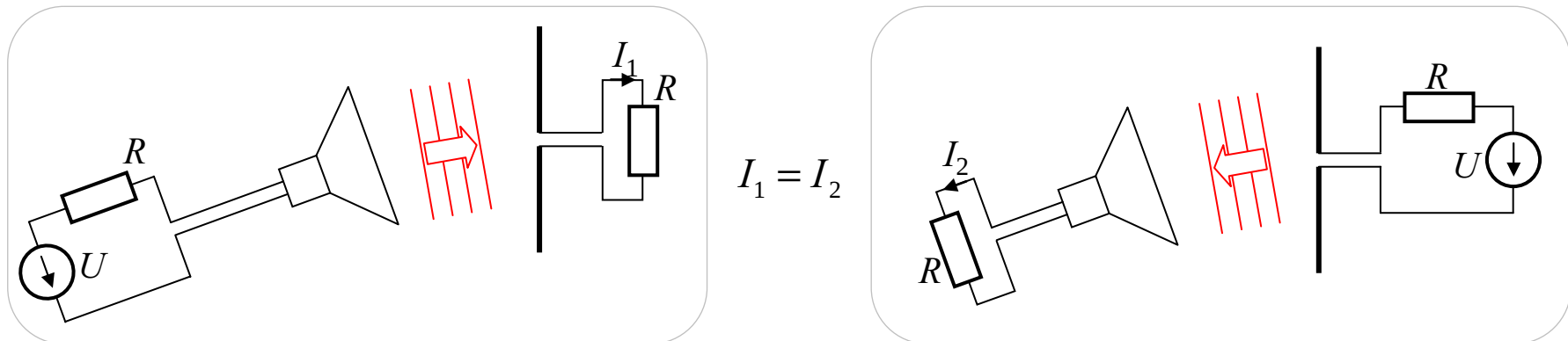
z.B. diskretes RLCÜ Netzwerk:

$$\int_V \vec{J}_1 \vec{E}_2 dV \rightarrow I_1^t U_2$$

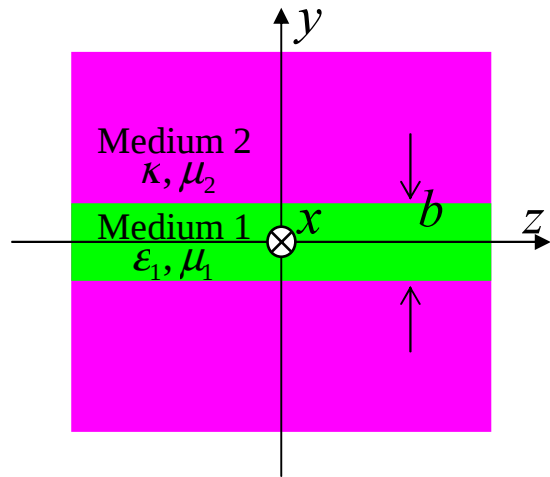
$$U = ZI$$

$$I_1^t U_2 = I_1^t (Z I_2) = I_2^t (Z^t I_1) \xrightarrow{\text{reziproke Impedanzmatrix: } Z^t = Z} I_2^t (Z I_1) = I_2^t U_1$$

Antennen:



Beispiel: TEM Welle 3 Schichten Problem



PEC Lösung:
$$H_x = \begin{cases} c_1 \exp(-jk_1 z) & \text{für } |y| \leq b/2 \\ 0 & \text{sonst} \end{cases}$$

$$E_y = \begin{cases} c_1 \frac{-jk_1}{j\omega\epsilon_1} \exp(-jk_1 z) & \text{für } |y| \leq b/2 \\ 0 & \text{sonst} \end{cases}$$

$$N = \text{Re} \left\{ \frac{1}{2} \int (-E_y H_x^*) dx dy \right\} = \frac{1}{2} |c_1|^2 \frac{k_1}{\omega\epsilon_1} \text{Re} \left\{ \int dy dz \right\} = \frac{1}{2} |c_1|^2 \frac{k_1}{\omega\epsilon_1} b \Delta x$$

$$P' = \frac{1}{2} \sqrt{\frac{\omega}{2}} \int \left(\sqrt{\frac{\mu_2}{\kappa}} |H_x|^2 \right) ds = \frac{1}{2} |c_1|^2 \sqrt{\frac{\omega\mu_2}{2\kappa}} \int ds = |c_1|^2 \sqrt{\frac{\omega\mu_2}{2\kappa}} \Delta x$$

$$\alpha = \frac{1}{2} \frac{P'}{N} = \frac{k_1}{2} \frac{\mu_2}{\mu_1} \frac{\delta}{b}$$

$$\delta = \sqrt{\frac{2}{\omega\mu_2\kappa}}$$