Universität Rostock — Traditio et Innovatio

Master's Thesis

Fast direct solver for electrostatic integral equations based on Haar wavelets

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System matrices arising from the discretization of integral equations are, in general, dense and the complexity of direct inversion is cubical (e.g., via Gauss elimination). Fast methods such as the multilevel fast multipole method or the adaptive cross approximation have been used to enable the construction of system matrices and the execution of matrix-vector products in quasi-linear complexity [1]. Together with iterative solvers such as conjugate gradient or GMRES solutions can, in the best case, be obtained in quasi-linear complexity.

However, in many practical scenarios iterative solvers converge only slowly or not at all due to ill-conditioning of the system matrix. This destroys the quasi-linear complexity set by the fast method and thus often direct solvers are preferred as they guarantee to obtain a solution.

Recently, fast direct solvers have been proposed (for a summary of the different methods, see the textbook [2]) and wavelets have been pointed out as one particular strategy [3]. This thesis shall investigate a fast solver strategy based on Haar wavelets for 3D electrostatic integral equations.

Tasks:

- Implement an integral equation solver for electrostatic problems;
- Construction of Haar wavelets;
- Fast construction of system matrix via Haar wavelets;
- Investigation of fast direct solution;
- Possibly comparison with other strategies.

Prerequisites: Solid mathematical background and programming skills.

References

[1] H. Harbrecht and R. Schneider, "Wavelet Galerkin Schemes for Boundary Integral Equations—Implementation and Quadrature," *SIAM Journal on Scientific Computing*, vol. 27, no. 4, pp. 1347–1370, Jan. 1, 2006.

[2] P.-G. Martinsson, *Fast Direct Solvers for Elliptic PDEs*, ser. CBMS-NSF Regional Conference Series in Applied Math 96. Philadelphia, PA: Society for Industrial and Applied Mathematics, 2020, 315 pp.

[3] H. Harbrecht and M. Multerer, "A fast direct solver for nonlocal operators in wavelet coordinates," *Journal of Computational Physics*, vol. 428, p. 110 056, Mar. 1, 2021.